Mon. Not. R. Astron. Soc. **000**, 000–000 (0000) Printed 1 February 2008 (MN LaTeX style file v1.4)

Thresholds on star formation and the chemical evolution of galactic discs: cosmochronology and the age of the galaxy

K. Chamcham[1,2] and M. A. Hendry[1]
[1] *Astronomy Centre, University of Sussex, Falmer, Brighton, East Sussex BN1 9QH*
[2] *University King Hassan II, Faculté des sciences Ain-Chok, B.P. 5366 Maarif, Casablanca, MOROCCO*

Accepted ——. Received ——; in original form 1 February 2008

ABSTRACT
In this paper we analyse different chronometers based on the models of chemical evolution developed in Chamcham, Pitts & Tayler (1993; hereafter CPT) and Chamcham & Tayler (1994; hereafter CT). In those papers we discussed the ability of our models to reproduce the observed G-dwarf distribution in the solar neighbourhood, age-metallicity relation and radial chemical abundance gradients. We now examine their response to the predictions of cosmochronology. We use the recent production ratios of the actinide pairs $^{235}U/^{238}U$ and $^{232}Th/^{238}U$ provided by Cowan, Thielemann & Truran (1991) and the observed abundance ratios from Anders & Grevesse (1989) to determine the duration of nucleosynthesis in the solar neighbourhood, and thus to determine maximum likelihood estimates and confidence intervals for the infall parameter, β, which controls the growth rate of the disc in our models. We compare our predictions for the age of the disc with the age of the galaxy estimated from models of white dwarf cooling and from the age of globular clusters. From our statistical analysis we find that these three methods of age prediction appear to be consistent for a range of maximum likelihood values of β which is in good agreement with the values considered in CPT and CT, which were found to give a good fit to the observational data examined in those papers. We also briefly consider the consistency of our results with the age of the universe predicted in different cosmological models – a topic which we will investigate more fully in future work.

Key words: cosmochronology – age of the universe – galaxy formation – disc instability

1 INTRODUCTION

In a series of earlier papers, models for the chemical evolution of the galaxy have been developed and studied by Chamcham, Pitts and Tayler based on the adoption of a threshhold – derived from the Toomre stability criterion – for the onset of star formation (SF) in the galactic disc. In Chamcham, Pitts & Tayler (1993; hereafter CPT) and Chamcham & Tayler (1994; hereafter CT) the ability of these models to reproduce the observed G-dwarf distribution in the solar neighbourhood, age-metallicity relation and radial chemical abundance gradients was discussed. In this paper we examine the response of our models to the predictions of cosmochronology for the age of the galactic disc.

The age of the galaxy continues to be one of the most debated subjects in astronomy – particularly in the light of a number of recent determinations of the Hubble constant (cf. Schmidt, Kirshner & Eastman 1994; Freedman et al. 1994) which appear to indicate an age of the *universe* of less than 10 Gyr for the standard inflationary model of a flat universe with zero cosmological constant. These results are in stark contradiction with astrophysical constraints on the age of the galaxy from, e.g., the cooling of white dwarfs and the age of globular clusters (cf. Renzini 1993). If both age determinations were correct then one would reach the absurd conclusion that the universe is younger than our galaxy, and the resolution of this paradox is one of the outstanding problems of observational cosmology today.

In Table 1 we summarise the constraints on the age of the universe reported in recent literature, as provided by the four methods of age determination which we consider in this paper. Of course, of these four methods, only by determining the values of cosmological parameters can one *directly* infer the age of the universe – and even then only in the context of a given cosmological model. As indicated in the first column of Table 1, the other three methods constrain the age of the solar neighbourhood, galactic disc and galaxy respectively – each of which may then be regarded as a (progressively larger) lower bound for the age of the universe. Thus, although the four methods are not directly comparable, one

can at least demand that their predictions should be physically consistent with each other – a useful approach which was adopted in, e.g., Tayler (1986) and which we also adopt in this paper. It would obviously be unreasonable for the age of the galaxy to be smaller than the age of the solar neighbourhood, or larger than the age of the universe, for example.

With respect to the second and fourth rows of Table 1, it would also be naive to imagine that all values in the cited range are equally likely. The age determined from cosmological models extends to the limits of the quoted range only by a somewhat delicate 'balancing act' between the values of the relevant cosmological parameters. In particular, if the Hubble constant is indeed high – say, $H_0 > 75$ – then all but the extreme lower limit of the quoted range is strongly excluded in the standard inflationary model. Similarly, if one believes the measured values and observational errors of the actinide pairs' abundance ratios which we consider in this paper, then we shall see that a careful statistical analysis can place tighter constraints on the most likely age range determined from cosmochronology – at least within the framework of a given chemical evolution model – than suggested by the full extent of the range quoted in Table 1.

In what follows we shall focus on the method of cosmochronology and compare its results with the predictions of the other methods. Cosmochronology has recently been gaining in credibility, especially with its extension to individual stars by Butcher (1987) – although applications have usually been limited to the solar system – and the progress of theoretical and experimental nuclear physics (Klapdor-Kleingrothaus 1991). This method is, however, still strongly dependent upon one's choice of chemical evolution model.

In CPT and CT we have shown that one of the weaknesses of our models is the number of free parameters which we are using. The aim of this paper is, therefore, to place some constraints on the most important of these parameters through the analysis of cosmochronometers – specifically, the abundance ratios of two actinide pairs – by confronting their predicted age of the Galactic disc with other astrophysical and cosmological predictions. The particular *strength* of our models, on the other hand, is that the star formation (SF) law is deduced consistently with some physical input, instead of the simple Schmidt law (e.g. SF proportional to the gas density) commonly used by different authors in their cosmochronological models (Schramm & Wasserburg 1970; Clayton 1988; Malaney & Fowler 1987; Pagel 1989).

Our present approach is analogous to that of Mathews and Schramm (1993) in that both models consider a delay in SF instead of an instantaneous birth of stars in the Galaxy. In our models this delay is related to the stability properties of the disc (cf. CPT), whereas in the model of Mathews & Schramm it is introduced only schematically to sketch the merger process in the formation of the Galaxy. In our models this delay varies between 1 – 5 Gyr at the solar neighbourhood, depending on the growth time of the disc (or equivalently the rate of infall of material onto the disc) and the relative roles of heating and cooling processes which regulate the star formation rate (SFR).

The structure of this paper is as follows. In section 2 we describe briefly our present chemical evolution model, summarising the relevant details from earlier papers. In section 3 we discuss the results of applying our chemical evolution model to predict the abundance ratios of the two chronometers as a function of the age of the disc, and compare these model predictions with the observed abundance ratios. We then describe a method for obtaining maximum likelihood estimates and confidence intervals for the infall parameter, β, which controls the growth rate of the disc in our models, as a function of the age of the disc. In section 4 we review in more detail the constraints on the age of the galaxy provided from white dwarfs and globular clusters. In section 5 we then discuss the consistency of the age predictions from our models with those from white dwarfs and globular clusters, and on this basis consider what contraints are placed on the parameter β, and on the age of the disc. We also briefly discuss the consistency of our results with the age of the universe predicted from the values of cosmological parameters – a subject which we will consider in more detail in a separate paper. Finally in section 6 we summarise our conclusions.

2 CHEMICAL EVOLUTION MODEL

2.1 Threshold of star formation

The determination of the age of the Galaxy by the method of cosmochronology provides at least a lower limit for the age of the universe. This method is based on the assumption that the earth and meteorites solidified some 4.5 Gyr ago and that the isolation and condensation of the solar system from the ISM occurred at $T_\odot = 4.6 \pm 0.1$ Gyr ago. The present-day abundance ratios of the chronometric pairs $^{235}U/^{238}U$ and $^{232}Th/^{238}U$ are determined from the analysis of meteorites (i.e. carbonaceous chondrites) and moon rock samples. Their values at the time of the formation of the solar system are extrapolated back in time and are respectively 0.33 ± 0.03 and 2.32 ± 0.23 (Anders & Grevesse 1989).

The duration of nucleosynthesis, $t_{\rm nuc}$, from the start of SF in the Galaxy until the formation of the solar system is determined by a suitable chemical evolution model. One can estimate from this a lower limit for the age of the galactic disc to be

$$T_D = T_\odot + t_{\rm nuc} \qquad (1)$$

More correctly, equation 1 accounts for the age of the solar neighbourhood – in other words the time since SF began at our location in the Galaxy. We thus rename this estimated age T_S instead of T_D. ⋆

The variation in the models of cosmochronology used by different authors stems from the different relations assumed between the SFR and the gas mass, and their choice of model for the infall of material onto the disc (Clayton 1988). To our knowledge all previous authors have assumed that SF begins either instantaneously everywhere in the Galaxy or – in the case of Mathews & Schramm (1993) – after an arbitrary delay of a few Gyr. As shown in the threshold models of CPT, however, the delay, δ, in the onset of SF in the solar neighbourhood is not an arbitrary parameter but is determined self-consistently by the other parameters of our chemical evolution model. We can therefore deduce a

⋆ Note that T_S is therefore comparable to the 'raw' age predicted by WD cooling.

model-dependent estimate for the age of the disc, T_D, given by

$$T_D = T_\odot + t_{\rm nuc} + \delta \tag{2}$$

In the chemical evolution models developed in CPT and CT (respectively without and with radial flows), there is no reference to the process of formation of the halo and therefore no prediction of the time delay between the formation of the halo and the disc. In other words, our chemical evolution model does not allow a direct estimation of the age of the halo, which we denote T_G, nor of course the age of the universe, which we denote T_U, since our model places no constraints on the 'halo gestation period' for our galaxy – i.e. the time elapsed from the Big Bang to the formation of the halo. As we already remarked in section 1, however, for *consistency* we clearly require that the following chain of inequalities always holds:

$$T_U > T_G \geq T_D \geq T_S \tag{3}$$

In this paper we will derive constraints on the infall parameter, β, of our chemical evolution model by comparing our model predictions for the age of the disc with other estimates of the age of the solar neighbourhood, the galactic halo and the universe, subject to the condition that the above inequalities should hold – in other words that our chemical evolution model should be consistent with other astrophysical and cosmological age determinations.

In order to determine the abundance ratios of the radioactive isotopes, we follow the same procedure as in CPT, with the simple difference that when SF starts, we solve the following equation for the evolution of the fraction of mass, Z_A, of a radioactive isotope A (Tinsley 1980)

$$\frac{dZ_A}{dt} = \frac{p_A s - aZ_A}{\Sigma_g} - \lambda_A \, Z_A \tag{4}$$

where λ_A is the rate of disintegration of the isotope, A, p_A is its yield – assumed to be constant – Σ_g is the gas surface density, s is the SFR and $a = a_0 f$ is the infall rate. We have used the model infall function

$$f = 5\beta \exp(-kr)[\mathrm{sech}(\beta \mathrm{t})(1 + \mathrm{l} \tanh(\beta \mathrm{t}))]/(1 + \frac{2\mathrm{l}}{\pi}) \tag{5}$$

where the denominator is introduced as a normalisation factor (to ensure that the final mass of the galaxy is not dependent on either β or l) and β is the infall parameter, as defined in CPT. We have chosen the value of the parameter a_0 such that the final mass of our model galaxy is in agreement with the observed range of values for our Galaxy. In conjunction with equation 4, we solve the equation for the gas surface density,

$$\frac{d\Sigma_g}{dt} = a - s \tag{6}$$

As in CPT, we require that SF starts only when the Toomre stability criterion is marginally satisfied, i.e.:-

$$Q = \frac{\kappa c_g}{\pi G \Sigma_g} \simeq 1 \tag{7}$$

where κ is the epicyclic frequency and c_g the gas velocity dispersion. This threshhold on SF implies the delay, δ, discussed above – which is partly dependent on the thermal properties of the disc and the characteristics of the infall, amongst other physical properties (See CPT for a more detailed discussion). In the present paper we concentrate on the properties of the infall rate.

We write equations 4 and 6 in their non-dimensional form

$$\frac{dz_A}{d\tau} = \frac{y - f \, z_A}{x} - \Lambda_A \, z_A \tag{8}$$

and

$$\frac{dx}{d\tau} = f - y \tag{9}$$

where we have used the notation defined in CPT; in particular here we have written $Z_A = p_A z_A$. With this notation, the abundance ratio of an isotopic pair (A,B) with a production ratio P_A/P_B is

$$\frac{N_A}{N_B} = \frac{P_A}{P_B}\frac{z_A}{z_B} \tag{10}$$

Here we have used the definition of the yield as the ratio of newly synthesised elements returned instantaneously to the ISM, to the fraction of mass locked up in long-lived remnants: i.e. $p_A = Am_u P_A/(1-R)$, where R is the ejection rate and m_u is the atomic mass unit. Furthermore we have defined $\Lambda_A = t_0 \, \lambda_A$ as the dimensionless disintegration rate. Note that a fully consistent analysis should ideally consider a variable yield, especially at the early stages of the production of the radioactive elements (cf. Pagel 1993).

In order to calculate the initial (i.e. at the time when SF begins) enrichment of a given isotope, A, we assume in our models that there existed a 'primordial' metallicity, z_{i0}, originating from earlier nucleosynthesis in the halo at an average time of approximately t_e Gyr before SF starts in the solar neighbourhood. This generic assumption can find some support in the observed high (≈ 0.5) [O/Fe] ratio for metal poor halo stars, indicating contamination from high mass stars formed during early SF in the galactic halo (Burkert 1993). Under this assumption the initial metal enrichment would then be given by $z_{Ai} = z_{i0} \cdot \exp(-\lambda_A t_e)$. To improve upon this approximation we would need to further improve the models of CPT by a detailed study of the early moments of galaxy formation, in order to be able to follow the chemical evolution of the halo and the halo/disc interaction. We will show in section 5, however, that the predicted abundance ratios at late times have negligible dependence on the adopted values of z_{i0} and t_e.

2.2 Radial flows

For the case where we have studied the effect of radial flows on predictions for the age of the disc, we have used the calculations developed previously in CT – with the only change in the metallicity equation, in which we have added the term $-\lambda_A \, Z_A \, \Sigma_g$ to the right hand side to account for the decay of the radioactive element in the gas. As was shown in CT, after a certain time of evolution of the disc when SF is well underway, incorporating radial flows in the model will tend to produce a lower metallicity. Hence for a *given* (i.e. the observed) metallicity one would expect the disc to appear *older* under the influence of radial flows than in their absence. This is because the fresh, metal-poor, gas flowing into the disc from the halo dilutes the heavy metals produced by SF.

The equation of evolution of radioactive elements is thus,

$$\Sigma_g\left(\frac{\partial Z_A}{\partial t}+v_r\frac{\partial Z_A}{\partial r}\right)=p_As-aZ_A-\lambda_A\Sigma_gZ_A \qquad (11)$$

Expressed in its non dimensional form this equation becomes

$$\frac{\partial z_A}{\partial\tau}+V_r\frac{\partial z_A}{\partial\bar{r}}=\frac{y-fz_A}{x}-\Lambda_Az_A \qquad (12)$$

where $\bar{r}=r/r_0$, $V_r=v_r/v_{0r}$, $v_{0r}=r_0/t_0$, with $r_0=1$ kpc and $t_0=1$ Gyr.

At this stage we need to treat carefully the solution of the above equation. When we include radial flows all regions of the disc are intercorrelated as star formation propagates outwards from the centre of the disc (see figure 1, where we plot the time, $t_\star$, at which SF begins at radius, $R_\star$) and ideally we should account for this when modelling the initial metal enrichment of the solar neighbourhood. A fully consistent treatment to this problem is far from trivial, however, and – as we indicated previously – would require a detailed model for the dynamical evolution of the halo, both prior to the onset of SF in the disc and throughout the subsequent evolution of the disc. We can, nevertheless, place useful limits on the impact of including radial flows on our age predictions for the disc by considering the case where the initial metal enrichment is identically equal to zero. Since the effect of radial flows will always be to dilute the primordial metallicity, z_{i0}, setting the initial enrichment equal to zero when SF begins at each radius in the disc is simply the limiting case of this dilution process.

3 RESULTS

We now describe in detail the numerical results of solving the equations of our chemical evolution models outlined in section 3. Our solution involves the adoption of numerical values for the following three parameters: K_1, K_2 and K_3, which denote the fraction of energy put into the ISM by infall, SF and cloud-cloud collisions respectively. We have used the following numerical values (arguments for the choice of which were given in CPT):

$$K_1=2\,,\quad K_2=25\,,\quad K_3=10 \qquad (13)$$

The values of the (dimensionless) disintegration constants which we adopt are $\Lambda_A\equiv0.0495$ for ^{232}Th, 0.985 for ^{235}U and 0.1551 for ^{238}U (cf. equations 8 and 12). Varying the remaining free parameters of our models, we found that the predicted abundance ratios are sensitive only to the infall parameter, β. We thus chose to address quantitatively the sensitivity of our age predictions to β and assigned to all other model parameters the same numerical values as in ‘Model 1’ of CPT (See Table 1 of CPT for further details).

Figure 2 illustrates the dependence of SF on the infall parameter, plotting the SFR as a function of time for $\beta=$ 0.1, 0.4, 0.7 and 1.0, and it is clear that the value of β determines not only the delay, δ, before the onset of SF, but also the duration and amount of SF in the solar neighbourhood, at a given time. For $\beta=0.1$ the onset of SF is delayed by $\delta\sim6$ Gyr, and SF then proceeds as a slowly varying function of time. For $\beta=1.0$, on the other hand, the delay, δ, is less than 1 Gyr, and is followed by a rapid burst of SF which has decayed to a few percent of its peak rate after only ~7 Gyr.

In order to address quantitatively this sensitivity to β, we consider below a statistical analysis designed to obtain maximum likelihood estimates of β, and thus constrain the law of SF, which ‘best fits’ our chemical evolution model to the observational age constraints in a manner consistent with the inequalities of equation 3.

Among other tests to which one can subject our models are their ability to solve the G-dwarf problem, to fit the age-metallicity relation and to predict the observed gas fraction in the solar neighbourhood. The relative success of the models in addressing these problems has already been discussed in CPT. In figure 3 we illustrate the explicit dependence of the gas fraction, μ, on the adopted value of β – associated with the SFR as shown in figure 2. We plot the model-predicted values of μ as a function of the present age of the galactic disc, and compare them to the observed value of $\mu_{\rm obs}=0.28\pm0.09$, taken from Gilmore, Kuijken & Wyse (1989), and denoted by the horizontal lines on figure 3. We also illustrate, shown on the upper ordinate of figure 3, the relationship between the model-predicted values of μ and the age of the *universe* – a lower limit for which is estimated by adding 1.5 Gyr to account for the time interval between the Big Bang and the formation of the galactic disc.

We can see from the figure that the predicted value of μ matches the observed value at a wide range of different disc ages, depending on the value of β, ranging from ~20 Gyr for $\beta=0.1$ to ~3 Gyr for $\beta=1.0$. N ote that for $\beta>0.4$ the best fit of the model predictions to the observations is found for a low disc age of $t<6.5$ Gyr – in good agreement with the age of the universe predicted for the standard inflation model with a fairly high ($H_0\sim70$) Hubble constant, even if our adopted disc ‘gestation period’ of 1.5 Gyr were something of an underestimate. The shape of the curves and the width of the observational error band on $\mu_{\rm obs}$, however, mean that considerably higher disc ages are not strongly excluded – even for $\beta=1$.

We have computed the abundance ratios of the chronometric pairs ^{232}Th/^{238}U and ^{235}U/^{238}U with the respective production ratios 1.6±0.1 and 1.24±0.1 from Cowan, Thielemann & Truran (1991). Our computations show that the quoted uncertainty of ±0.1 on the production ratios introduce a spread of about ±1 Gyr on the predicted age, in the sense that the higher the production ratio the younger the disc and vice-versa.

Figures 4a and 4b show the time evolution of the model predicted values of the abundance ratios of our two chronometers – again associated with the same SFR of figure 2. As in figure 3, the observed abundance ratio and its 1σ error band are denoted by the horizontal lines on each figure. These graphs have been obtained adopting a delay of $t_e=3$ Gyr between the period of nucleosynthesis in the halo – which we assume resulted in a ‘primordial’ metallicity of $z_{i0}=0.25$; see section 2 – and the onset of SF in the solar neighbourhood. Reasons for the choice of $z_{i0}=0.25$ are discussed in CPT; we show in section 5, however, that the adopted values of both z_{i0} and t_e have negligible impact on our results.

Note that, in a similar manner to figure 3, we estimate the time evolution of the production ratios in our model as a function of the age of the universe (as shown in the upper

ordinates of figures 4 and 5) by adding 6 Gyr to the age of the disc: a disc gestation period of 1.5 Gyr and a further 4.5 Gyr representing the interval from the formation of the solar system to the present day – during which time the abundance ratios have been 'frozen'. As in the case of figure 3, this second age scale is designed to be largely illustrative – simply in order to allow an approximate lower bound for the age of the universe to be estimated from our models.

A quantitative examination of figures 4a and 4b reveals several interesting features. Firstly, note that the initial value of the ^{232}Th/^{238}U ratio predicted in our model lies within the 1σ error band of the observed value of this ratio, and only deviates significantly from the observed ratio several Gyr after the onset of SF (the exact time depending on the value of β) before reaching a minimum value and then increasing monotonically at later times. The ^{232}Th/^{238}U abundance ratio, on the other hand, is predicted in the model to have an initial value several standard deviations below the observed ratio, to increase rapidly after the onset of SF, to reach a maximum several Gyr later and then to decay monotonically at later times.

A naive interpretation of these figures could then lead to the conclusion that our model predictions give a good fit to the observed abundance ratios for very young ages. For example, in the case of $\beta = 0.4$ we see from figure 4b that the ratio ^{232}Th/^{238}U rises sharply to intercept the observed abundance ratio when the age of the disc is only 2 Gyr. At the corresponding age on figure 4a we see that the model predicted ratio of ^{232}Th/^{238}U has decreased only very slightly from its initial value, and still lies comfortably within the 1σ error band. Thus, when $\beta = 0.4$, it is apparent that the observed and predicted abundance ratios are in good agreement for a disc which is only 2 Gyr old. The obvious flaw in this interpretation, however, is the fact that SF has begun only 1 Gyr before the formation of the solar system with these model parameters. In other words this apparently good fit to the observed abundance ratios does not allow a nearly sufficient duration for nucleosynthesis. We know, from the ages of WD in the solar neighbourhood, that significant amounts of SF had occured in our region of the galactic disc around 9 Gyr ago[†]. Allowing 4.5 Gyr for the age of the solar system, this means that we should allow a minimum of ~ 4.5 Gyr to account for the duration of nucleosynthesis - i.e. the interval, $t_{\rm nuc}$, between the onset of SF in the solar neighbourhood and the formation of the solar system. In particular, therefore, any assessment of the 'goodness of fit' between our model predictions and the observed abundance ratios must ensure that a sufficiently long period of nucleosynthesis is allowed. Similarly, a statistical analysis designed to estimate 'best fit' values of the infall parameter, β, should exclude all cases in which the duration of nucleosynthesis is unreasonably short. We now describe such a statistical analysis.

We begin by introducing some notation. Let $\mathrm{U_{obs}}$ and $\mathrm{Th_{obs}}$ denote the observed abundance ratios of ^{235}U / ^{238}U and ^{232}Th / ^{238}U respectively, and let $\mathrm{U_{true}}$ and $\mathrm{Th_{true}}$ denote the true values of these ratios. We define $\varepsilon_{\rm U}$ and $\varepsilon_{\rm Th}$, the observational errors on the measured values of the production ratios, as:

$$\varepsilon_{\rm U} = \mathrm{U_{obs}} - \mathrm{U_{true}}$$

$$\varepsilon_{\rm Th} = \mathrm{Th_{obs}} - \mathrm{Th_{true}}$$

Finally we define $\mathrm{U_{model}}(t,\beta)$ and $\mathrm{U_{th}}(t,\beta)$ as the present day values of ^{235}U / ^{238}U and ^{232}Th / ^{238}U predicted in our model as a function of the age of the universe, t, and the value of the infall parameter, β. We wish to construct a joint likelihood distribution, $\mathcal{L}(t,\beta)$, for t and β under the null hypothesis that our model is correct. In other words, $\mathcal{L}(t,\beta)$ measures the probability that one would obtain the observed values of the abundance ratios, given that their true values are equal to the values predicted in our model – for given t and β. Thus, under the null hypothesis, we have:

$$\varepsilon_{\rm U} = \mathrm{U_{obs}} - \mathrm{U_{model}}(t,\beta)$$

$$\varepsilon_{\rm Th} = \mathrm{Th_{obs}} - \mathrm{Th_{model}}(t,\beta)$$

We assume that $\varepsilon_{\rm U}$ and $\varepsilon_{\rm Th}$ are normally distributed with zero mean and dispersion equal to the standard error of the respective abundance ratio, as reported in the literature. We also assume that $\varepsilon_{\rm U}$ and $\varepsilon_{\rm Th}$ are uncorrelated - which is certainly not the case for the values of $\mathrm{U_{true}}$ and $\mathrm{Th_{true}}$, but is likely to be a reasonable approximation for their observational errors.

The joint likelihood function, $\mathcal{L}(t,\beta)$, is then given by:

$$\mathcal{L}(t,\beta)dtd\beta = A\exp\left[-\frac{1}{2}\left(\frac{\mathrm{U_{obs}}-\mathrm{U_{model}}(t,\beta)}{\sigma_U}\right)^2 - \frac{1}{2}\left(\frac{\mathrm{Th_{obs}}-\mathrm{Th_{model}}(t,\beta)}{\sigma_{Th}}\right)^2\right] S(t,\beta)dtd\beta \qquad (14)$$

Here $\sigma_{\rm U}$ and $\sigma_{\rm Th}$ denote the dispersion of $\varepsilon_{\rm U}$ and $\varepsilon_{\rm Th}$ respectively, A is a normalisation constant and $S(t,\beta)$ is a selection function which excludes values of t and β which are physically unreasonable due to, e.g., too short a duration for nucleosynthesis.

It now follows from equation (14) that the likelihood distribution of t conditional upon β is given by

$$\mathcal{L}(t|\beta)dt = \frac{\mathcal{L}(t,\beta)dt}{\int \mathcal{L}(t,\beta)dt} \qquad (15)$$

In figure 5 we plot this distribution for the illustrative values of $\beta = 0.1, 0.4, 0.7, 1.0$, as before. We assume a *minimum* duration, $t_{\rm nuc}$, of nucleosynthesis of 4.5 Gyr. Of course the minimum acceptable age for the disc at the formation of the solar system is in practice higher than 4.5 Gyr, since our model also predicts a β dependent time delay, δ, before the onset of SF in the solar neighbourhood, as we have already seen in figure 2, and this limiting age causes the slight skewness in the likelihood distribution for $\beta = 0.7$ and 1.0 in figure 5. We see that for $\beta = 0.1$ we require an age for the disc at solar system formation of between ~ 13 Gyr and ~ 24 Gyr to match the observed abundance ratios with a non-negligible likelihood, while for $\beta = 1.0$ the likelihood is non-negligible over the much narrower range of $\sim 5.3 - 7$ Gyr. Note that, while the likelihood distributions for $\beta = 0.1$ and 0.4 are effectively disjoint, there is a significant overlap between the likelihood distribution for $\beta = 0.4$ and 0.7 and for $\beta = 0.7$ and 1.0. This reflects the fact that the likelihood distributions become increasingly 'pushed up' against the lower limit of $t_{\rm nuc} \geq 4.5$ Gyr as β increases. Another consequence of this is the fact that as t decreases, the value of

† See also section 4 for a more detailed discussion

β is less well constrained by the observed abundance ratios, since there is a wider range of values of β for which $\mathcal{L}(t|\beta)$ is comparable in magnitude.

We can illustrate more precisely the constraints on β as a function of t by considering the conditional likelihood, $\mathcal{L}(\beta|t)$, given by

$$\mathcal{L}(\beta|t)d\beta = \frac{\mathcal{L}(t,\beta)d\beta}{\int \mathcal{L}(t,\beta)d\beta} \quad (16)$$

Figure 6 shows likelihood curves for $\mathcal{L}(\beta|t_{\rm disc})$ for several values of $t_{\rm disc}$, the age of the disc at the formation of the solar system. It is clear that the likelihood distribution does indeed become slightly narrower as the age of the disc increases. Using equation 14 it is straightforward to construct a curve of the maximum likelihood estimate, $\hat{\beta}_{ML}(t_{\rm disc})$, of β as a function of $t_{\rm disc}$. We plot this curve in figure 8. We can see that if $t_{\rm disc}$ lies in the range e.g. 8 - 12 Gyr, this corresponds to a maximum likelihood estimate of β lying in the approximate range 0.25 - 0.45.

Of course the form of the likelihood curves plotted in figures 5 – 7 depends in part upon our adopted value for the minimum duration of nucleosynthesis – so far taken to be 4.5 Gyr. Increasing $t_{\rm nuc}$ to, e.g., 6 Gyr causes a further increase in the skewness of the likelihood distributions, $\mathcal{L}(t|\beta)$ for large values of β. The likelihood distributions conditional upon age, $\mathcal{L}(\beta|t)$, are largely unaffected by the increase in $t_{\rm nuc}$, however, although of course there is now no likelihood curve for a disc age of 6 Gyr as this value is excluded by the higher value of $t_{\rm nuc}$ because one must also allow for the β dependent time delay before the onset of SF in the solar neighbourhood. Figure 8 shows the maximum likelihood estimate of β as a function of $t_{\rm disc}$, but now with $t_{\rm nuc}$ constrained to be at least 6 Gyr. We see that there now exists no acceptable solution for $\hat{\beta}_{ML}(t)$ for a disc younger than $\sim$ 7 Gyr. In other words, it is not possible for our model to build the disc quickly enough and still allow at least 6 Gyr between the onset of SF and the formation of the Solar System. Adopting our estimate of 1.5 for the disc gestation period, this translates into a lower limit on the age of the universe, T_U, of 13 Gyr. If T_U lies in the interval $\sim 13-13.5$ Gyr, the effect of increasing the lower limit on $t_{\rm nuc}$ to 6 Gyr is to force a sharp increase in the value of $\hat{\beta}_{ML}(t)$, in order that SF begins *early* enough to give a sufficiently long duration for nucleosynthesis. Above an age of $\sim$ 13.5 Gyr, however, $\hat{\beta}_{ML}(t)$ is identical to the value obtained with the smaller lower limit of $t_{\rm nuc} \geq 4.5$ Gyr, as shown in figure 7 – a fact which is easily verified analytically from differentiation of equation 16.

Following the method described in Hendry & Simmons (1990) we can construct confidence curves for the infall parameter, β, and the age of the disc, $t_{\rm disc}$, when the solar system formed. From these curves we can derive confidence intervals for β given $t_{\rm disc}$, or $t_{\rm disc}$ given β. Examples of such curves are shown in figures 9a and 9b, where the confidence intervals are derived at 67% and 90%, and with an assumed lower limit of $t_{\rm nuc} \geq 4.5$. By drawing a vertical line at some fixed value of $t_{\rm disc}$ the points at which the line intercepts the two curves define the limits of the appropriate confidence interval for β.

We can see from figures 9a and 9b that as $t_{\rm disc}$ increases, the confidence intervals for β become somewhat narrower in width and the limits of the intervals are monotonically decreasing – i.e. the estimated value of β is both smaller and more tightly constrained for larger values of $t_{\rm disc}$. We can see, moreover, that for $t_{\rm disc} < 8$ Gyr, the confidence intervals are pushed sharply to high values of β. This effect is also dependent on the adopted lower limit for $t_{\rm nuc}$, and can be seen to be somewhat more pronounced for the confidence curves with $t_{\rm nuc} \geq 6$ Gyr, as shown in figures 10a and 10b. Clearly a high value of β is required if SF is to begin in the solar neighbourhood quickly enough to allow a disc age of less than 8 Gyr when the solar system is formed and at the same time allow a sufficient duration for nucleosynthesis – particularly with $t_{\rm nuc} \geq 6$ Gyr. As was the case with figures 7 and 8, on the other hand, the adopted lower limit on $t_{\rm nuc}$ has no effect on the shape of the confidence curves for $t_{\rm disc} > 10$ Gyr.

In order to constrain β more precisely from this analysis – and to determine if the age predictions of our model can be made consistent with the inequalities of equation 3 – we now consider what are the constraints upon the age of the disc from estimates of white dwarf cooling and globular clusters.

4 CONSTRAINTS ON THE AGE OF THE GALACTIC DISC FROM OTHER METHODS

4.1 White dwarfs

On the basis of the theory of cooling of white dwarfs (WD), developed by Mestel (1952), Schatzman (1958) and Mestel & Ruderman (1967), which gives a relation between their age and luminosity, one can estimate the age of the galactic disc by measuring the luminosity of the coolest white dwarfs (Schmidt, 1959). This method requires the adoption of an equation of state for white dwarfs, knowledge of their chemical composition (i.e. the relative abundance of C, O, He and H) and the fluid – crystal phase transition, since the cores of white dwarfs are supposed to crystallize at low luminosity. We do no more than summarise some salient points from recent applications of this method. For a comprehensive review see e.g. D'antona & Mazzitelli (1990).

Hotter WD cool more rapidly, therefore the space-density of WD is expected to increase monotonically with decreasing WD luminosity. On the other hand, because of the finite age of the galactic disc there should be a paucity of WD at low luminosity. This is confirmed by observations which show an abrupt fall off in the number of WD stars below $\log(L/L_\odot) = -4.5$. Therefore, because the WD are considered to be the oldest stars in the disc, the age of the galactic disc can be inferred by computing the time required for them to cool to this luminosity, with a correction to account for their main sequence (MS) lifetime.

Winget et al. (1987) have fitted the observational data of WD luminosities with a model assuming a pure carbon core and a constant birthrate of WD over the age of the Galaxy. They deduced an age of the galactic disc $T_{WD} = 9.3 \pm 2$ Gyr. They then adopted an estimate for the age of the universe of $T_U = 10.3 \pm 2.2$ Gyr, which they claim accounts for the "time between the Big Bang and the first appearance of stars in the Galactic disk". It seems to us that their addition of only 1 Gyr to arrive at an estimate of T_U is at best somewhat arbitrary and at worst a considerable underestmate, given that their T_{WD} is inferred only from WD

in the solar neighbourhood and hence only takes account of the history of SF at our location in the disc. Their value of $T_{WD} = 9.3 \pm 2$ Gyr *does*, however, provide a reasonable estimate of the age which we denote by T_S in equation 3 – the age of the solar neighbourhood.

Iben & Laughlin (1989) considered in their model more ingredients, such as the IMF, time-variable SFR and the explicit influence of the lifetimes of MS progenitors. They found that the shape of the luminosity function is independent of any variation in the SFR, supporting thus the assumption of Winget et al. They deduced an age for the galactic disc of $T_{WD} = 9 - 10$ Gyr, consistent with the Winget et al. result.

As with the Winget et al. study, we regard this result as a reliable estimate of T_S. Both determinations should be regarded as no more than a lower limit for the present age of the galactic disc, T_D, and the age of the galaxy, T_G, for the following reasons:

(i) disc stars in the solar neighbourhood form certainly after the halo and bulge stars with a time delay of a few Gyr
(ii) not all the stars born in the disc stay there, especially the first generation of stars which inherit high random velocities from the gas out of which they formed. These stars therefore undergo more scattering, contributing thus to the WD paucity observed
(iii) any peculiar phenomenon liberating energy (nuclear, gravitational etc.) will tend to prolong the cooling time, and therefore increase the age of the galactic disc obtained by this method.
(iv) there is no reason why the evolution of WD should be independent of the chemical evolution of the disc, a fact pointed out in Pitts & Tayler (1992). In fact Pitts & Tayler showed, from their chemical evolution model, that the study of WD may be an underestimate of the true age: they deduced an age ~ 1 Gyr higher than that predicted by Winget et al.

4.2 Globular Clusters

Globular clusters (GC) are known to be the oldest observed stellar systems in our Galaxy – and to some extent in the Magellanic Clouds. All studies have generally converged upon an age in the range $T_{GC} = 13 - 15 \pm 3$ Gyr (VandenBerg 1990; Renzini 1993), although the age probability distribution is highly skewed and renders ages below 12 Gyr very unlikely (Tayler 1986, Renzini 1993).

The age of GC can be determined by comparing their observed H-R diagrams with theoretical H-R diagrams calculated as a function of time. Assuming that all the observed stars in the cluster have been formed on a time-scale which is short compared to the cluster age, and that all stars are formed from matter of the same composition, a direct measure of the age of the cluster can be obtained if the luminosity of the MS stars of given composition is known at the turnoff point (TO) – the position on the HR diagram which nearly coincides with the exhaustion of H at the centre of evolving stars – by using the time-luminosity relation for the MS-TO (VandenBerg 1988). The two methods commonly used to obtain the TO luminosity involve one of the following:

(i) the direct MS fitting of observed colour-magnitude diagrams to theoretical models
(ii) the calibration of the magnitude difference between H-B stars and the MS turnoff, at the colour of the turnoff.

The accuracy of either method depends upon how realistic is the theoretical H-R diagram used (Chiosi, Bertelli & Bressan 1992), as well as the extent of the uncertainties relating to the observed chemical compositions, reddening and distance moduli (Renzini 1993). It is these systematic effects which chiefly contribute to the quoted systematic uncertainty of ± 3 Gyr in Renzini (1993).

The GC 47 Tucanae has been the most studied and thus has the most reliably determined age (Hesser, Harris & VandenBerg 1987). Assuming an initial helium content of $Y = 0.24$ for the GC stars, its age was found to be $T_{47\mathrm{Tuc}} \simeq 13.5 \pm 2$ Gyr. Note that the determination of the age of the GC is very sensitive to the uncertainty in the He abundances, whose minimum values should be bound by the primordial Big Bang abundance (Tayler 1986). VandenBerg argues that an uncertainty as small as $\Delta Y = \pm 0.02$ has considerable ramifications for GC ages. There is no clear age-metallicity relation for GC, however, even if the choice of the metallicity seems to have a great influence on the determination of the age – as shown in the case of the GC M92. Adopting a composition of $Y = 0.24$, $[Fe/H] = -2.03$ and $[O/Fe] = 0.70$ for M92 yielded an estimated age of $T_{92} \sim 14$ Gyr. Adopting $[Fe/H] = 0$, with the other ratios unchanged, gave $T_{92} \sim 17$ Gyr. One is forced to conclude that the absolute value of the age depends critically on the assumed chemical composition of the member stars. Hence any improvement in GC age determination relies, in part, on improved determinations of their chemical composition. Another important source of uncertainty is the stellar evolutionary model adopted. As an example, Vandenberg has shown that the inclusion of Helium diffusion – which has been shown to be an important process in Pop II stars – in the canonical models of stellar evolution can reduce the age of the GC by about 15% – pushing back towards the age predicted by WD.

Chaboyer (1995) has also attempted to study the influence on the GC age determination of the uncertainties in the input physics used in stellar evolution models, and concluded that the spread in 'acceptable' GC age determinations – given the systematic errors – could be at most 11 - 21 Gyr. The two major sources of error identified in this study originate from the difficulty of modelling convection in stellar evolution models (e.g. changing the mixing length from 1.5 to 3 added an uncertainty in the age of up to 10%), and from the uncertainties in distance determinations to GC. Currently, distance moduli cannot be determined to an accuracy of better than ± 0.2 magnitudes, which contributes an uncertainty of up to 22% to the inferred GC age (Renzini 1993). Improvements in the calibration of each of the distance indicators commonly used – RR Lyrae variables, Subdwarfs and WD – are expected to come from the HST and the Hipparcos satellite in the near future, however.

In summary, if we assume that GC are of a comparable age to that of the galaxy itself, then GC ages essentially provide a direct measure of T_G in equation 3. Following Renzini (1993) and Chaboyer (1995), we adopt the estimate $T_G = 13 - 15 \pm 3$ Gyr for the purposes of comparison with

our chemical evolution models – with the important proviso that ages of *less* than 12 Gyr are highly unlikely.

5 DISCUSSION

We now turn to the main question of this paper: how do the predictions of our chemical evolution models fit into the picture of other predictions for the age of the disc, and what constraints do they allow us to place on the value of β?

We have seen in figures 7 and 8 that the maximum likelihood value of β decreases monotonically as the age of the galactic disc increases. Suppose that the age of the disc at the formation of the Solar System is taken to be 9 Gyr – which would imply an estimate for T_U of $\sim$ 15 Gyr, assuming a reasonably short gestation period of 1.5 Gyr for the disc. This value would also be in agreement with the age predictions of globular clusters discussed above. In this case the maximum likelihood estimator of β, $\hat{\beta}_{\rm ML} = 0.39$ (cf. figure 8), and a 67% confidence interval for β is found from figure 9a to be $0.34 \leq \beta \leq 0.46$ (assuming $t_{\rm nuc} \geq 4.5$ Gyr; the confidence interval is very slightly wider if we require $t_{\rm nuc} \geq 6.0$ Gyr). We can see from figure 2 that, for this maximum likelihood value of β, SF began in the solar neighbourhood about 2 Gyr after the formation of the disc – i.e. about 11.5 Gyr ago, which is also perfectly consistent with the constraints on T_S from WD cooling. We can see, moreover, from figure 3 that $\beta = 0.4$ gives a reasonable fit to the observed gas fraction, for $T_D = 13.5$ Gyr. Recall also from CPT that this value of β was successfully able to fit the observed age-metallicity relation and to solve the G-dwarf problem. Thus, a value of $\beta \simeq 0.4$ would seem to be favoured by our models – predicting as it does a disc age which is consistent with the acceptable ranges for the other disc age predictions which we have considered.

The biggest difficulty in achieving consistency with the inequalities of equation 3 when $\beta = 0.4$ would, therefore, seem to be finding a viable cosmological model which allows $T_U \geq 15$ Gyr. This is not an easy task, however, if H_0 lies in the range of 60 – 70 km s^{-1} Mpc^{-1}, as a number of recent reliable determinations have reported (c.f. Freedman et al. 1994; Schmidt, Kirschner & Eastman 1994; Riess, Press & Kirschner 1995). Such a high value of H_0 would certainly rule out the standard inflationary universe, with $\Omega_0 = 1$ and $\Lambda_0 = 0$. Adopting the lower bound for the matter density, $\Omega_{\rm m} \geq 0.35$, deduced in Liddle et al. (1995), we find that we *can* have $T_U \sim 15$ Gyr provided $\Lambda_0 \neq 0$, an issue previously examined in Tayler (1986) in which a similar conclusion was reached. Thus, it would appear that constraints on cosmological parameters still allow the consistency of *all* of the inequalities in equation 3 for a value of $\beta \simeq 0.4$, although certainly the range of compatible values of H_0, Ω_0 and Λ_0 is severely limited. In Hendry & Chamcham (1995, in preparation) we investigate in detail the current observational status of these cosmological parameters, and address specifically the issue of how readily one can match the consistent age predictions for the galactic disc which we have discussed in this paper with the age of the universe predicted in cosmological models.

We now consider the sensitivity of our results to the adopted value of the primordial metallicity, z_{i0}. Figure 11 shows the 67% confidence curves for β and the age of the disc, assuming $t_{\rm nuc} \geq 4.5$ Gyr and now with $z_{i0} = 0$. We can see immediately, on comparison with figure 9a, that for *all* values of β the change in the confidence curves is negligible. Specifically, the confidence limits for the age of the disc at a given value of β are increased by $\sim$ 0.1 Gyr (or equivalently the confidence limits for β at a given disc age are increased by $\sim$ 0.01) when z_{i0} is decreased from 0.25 to the limiting value of zero. Although the magnitude of the difference in the confidence limits between figures 9a and 11 is clearly negligible, it is worth noting that the *sign* of the difference is certainly in accordance with our physical understanding: when the primordial metallicity is reduced, one would expect to need a slightly longer time to build the observed heavy element abundance ratios, for a given value of β.

Note that figure 11 also demonstrates that our results are completely insensitive to the adopted value of t_e: setting $z_{i0} = 0$ is numerically equivalent to letting t_e tend to infinity. Although very large values of t_e would, of course, not make physical sense – as in any case t_e is constrained to be shorter than the time interval between the Big Bang and the onset of SF in the disc – it is, nevertheless, clear that the chosen value of t_e is essentially unimportant to our analysis.

Finally we consider the effect on our results of including radial flows in our chemical evolution models. As we explained in section 2, a completely self-consistent treatment of this problem would require a detailed model for the dynamical evolution of the halo. Instead, in this paper, we again consider the limiting case where $z_{i0} = 0$ everywhere in the disc at the onset of SF. Thus we explicitly include the effects of radial flows in the solution of equation 12 only *after* SF begins, which is essentially equivalent to assuming that the inflow of fresh, metal-poor, gas from the halo prior to the onset of SF *completely* dilutes the metal content of the disc during that time. Figure 12 shows the 67% confidence curves for β and the age of the disc in this case, and assuming $t_{\rm nuc} \geq 4.5$ Gyr. We can see from this figure that, for a disc age of 9 GYr at the formation of the Solar System, the 67% confidence interval for β is now $0.40 \leq \beta \leq 0.52$ – i.e. the upper and lower confidence limits for β are increased by 0.06.

This small positive shift in the confidence interval for β is again in accordance with our physical expectations: in the radial flow model, the continual dilution of the disc with metal-poor gas from the halo would imply that we need a slightly higher value of β in order to build the observed heavy element abundances in a given time. The shift is clearly not a large effect, however, particularly since a more realistic model for the dynamical evolution of the halo prior to SF in the disc would always result in *less* dilution of the disc metallicity by radial flows, thus resulting in an even smaller shift in the confidence curves. Hence, it seems that a value of β close to, or very slightly larger than, 0.4 is the most favoured in order to achieve a consistent picture with the age predictions of white dwarfs and globular clusters.

6 CONCLUSIONS

In order to place constraints on the most sensitive parameter of our chemical evolution models, we have compared in this paper the abundance ratios of heavy actinide pairs predicted by the threshhold galactic disc models of CPT

and CT with their currently observed abundance ratios in the Solar System. We have assumed that these nuclei are produced in direct proportion to the rate of star formation and calculated their abundances as a function of time, allowing for their decay. The values of the abundance ratios $^{235}U/^{238}U$ and $^{232}Th/^{238}U$ at the time of formation of the solar system are well known and the aim of our calculations has been to discover at what disc age the observed ratios are attained. We have studied a series of disc models with all parameters fixed except one: the parameter β which determines the rate of infall of matter to the disc. We also carried out some calculations varying the initial metallicity of the disc, produced by earlier stars in the halo, but found that this factor had negligible influence on the ages predicted by our models. Thus it seems that – at least in the context of our models – the heavy element abundances in the Galactic disc are determined essentially by the stellar component of the disc itself, and are independent of the history of the halo.

In view of other evidence on the age of the oldest nearby stars, we have required that star formation began in the Solar neighbourhood at least 9 Gyr ago, and have incorporated this constraint into a statistical analysis designed to determine our maximum likelihood estimates β as a function of the current age of the galactic disc, and vice versa.

Our model predictions showed that the deduced age of the Universe is a monotonically decreasing function of β. A high value of β implies both rapid disc formation and a strong initial burst of star formation. A value of $\beta \simeq 0.4$ predicts the current age of the galactic disc to be ~ 13.5 Gyr, which is consistent with current estimates of globular cluster ages, and is also in agreement with the models considered in CPT which satisfy other constraints on the properties of the local galactic disc – such as the G-dwarf distribution, the age metallicity relation and the fraction of gas content. Lower values of β require a very large disc age to satisfy the cosmochronological contraints.

We have also studied the effect on the age predictions for the galactic disc of incorporating radial flows in our chemical evolution models, following the treatment presented in CT. We simplified the analysis by assuming that the initial enrichment was reduced to zero everywhere in the disc in our models, We found that, even in this limiting case, the 67% confidence limits for β at a given disc age were increased by only ~ 0.05; we argued that any realistic, self-consistent model for the dynamical evolution of the halo and the disc under radial flows would result in *less* dilution of the disc metallicity, and thus result in a smaller positive shift in the confidence intervals for β.

We have also considered the consistency of the age determined by cosmochronology with other constraints on the age of the galaxy. The estimated disc age for $\beta = 0.4$ is larger than that deduced from white dwarf cooling, but within the uncertainties of that age. Our prediction is in any event perfectly *consistent* with the ages of white dwarfs in the Solar neighbourhood since, as pointed out by Pitts and Tayler (1992), a delay in the onset of star formation means that the age deduced locally from white dwarfs is somewhat less than the true disc age.

This leaves as the only major problem a reconciliation of the disc age with the age of the Universe obtained from current estimates of cosmological parameters. Of course this problem is essentially no different to that of reconciling globular cluster ages with cosmology. If the value of the Hubble constant is indeed greater than 60kms^{-1}Mpc^{-1}, as recent observations suggest, then to achieve consistency between the age of the universe and *both* our predicted age of the disc and the age of globular clusters seems to require a non zero cosmological constant. If globular cluster ages could be reduced to as little as 11 Gyr – which has been proposed by Chaboyer (1995) as a very robust lower limit, and which would again make viable a cosmological model with $H_0 = 60$ and $\Lambda_0 = 0$ – then a higher value of β might be more appropriate for our chemical evolution models. A value of $\beta = 1$ can give a local disc age comparable to that of white dwarfs – indicating that star formation began in the Solar neighbourhood within 1 Gyr of the formation of the disc – and an age of the Universe of 11 - 12 Gyr. Such a model would imply the presence of luminous discs at much higher redshift than in the case of $\beta = 0.4$ and this would therefore provide a suitable observational test to discriminate between these two cases. We will investigate high β models, together with a more detailed study of the current status of cosmological parameters, in a forthcoming paper.

ACKNOWLEDGEMENTS

The authors would like to thank Roger Tayler for many helpful discussions, and acknowledge the use of the STARLINK computer facilities at the University of Sussex. MAH acknowledges the hospitality of SISSA, where much of this work was carried out. MAH was supported by a PPARC personal research fellowship. KC gratefully acknowledges the award of an IAU travel grant in connection with this research.

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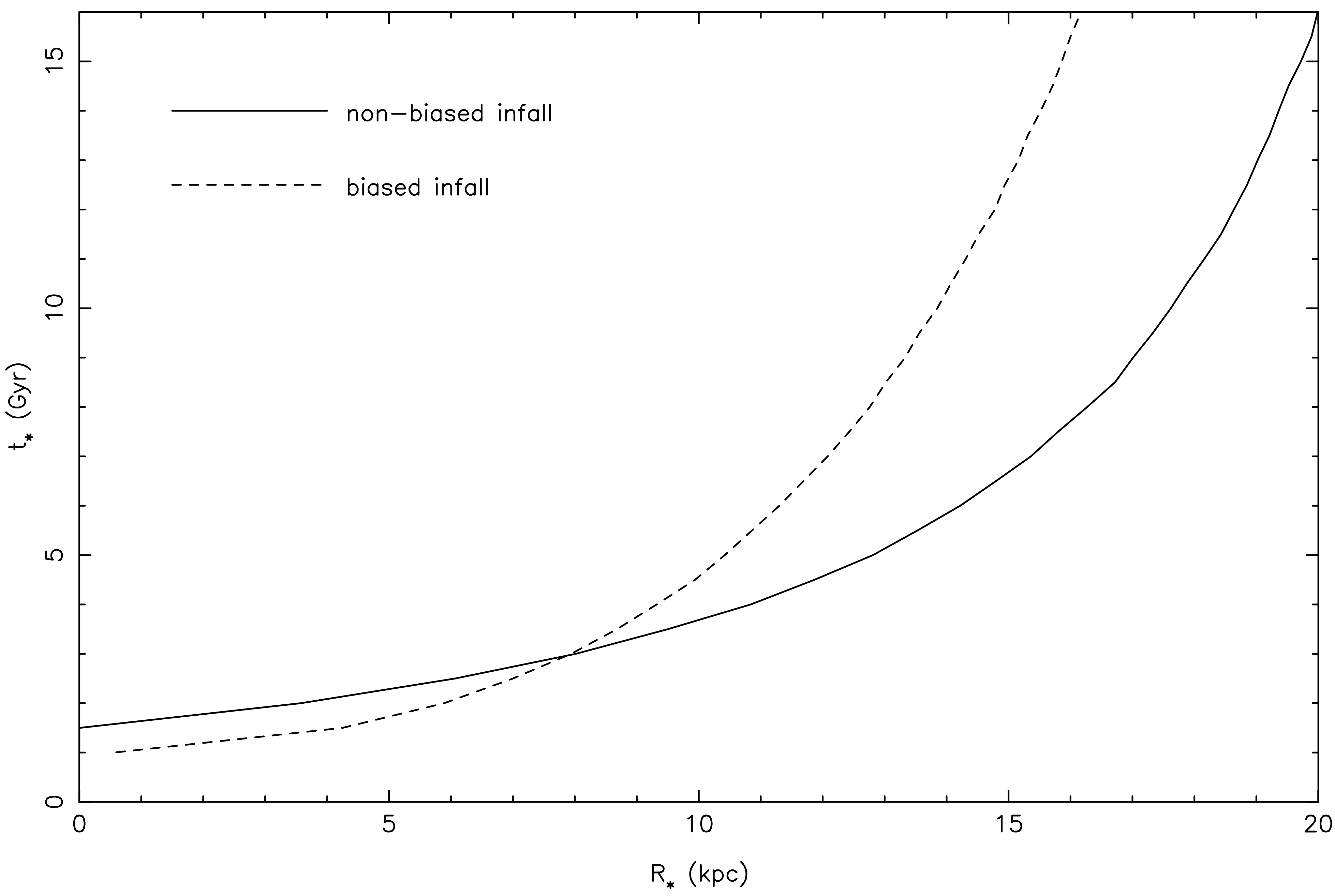
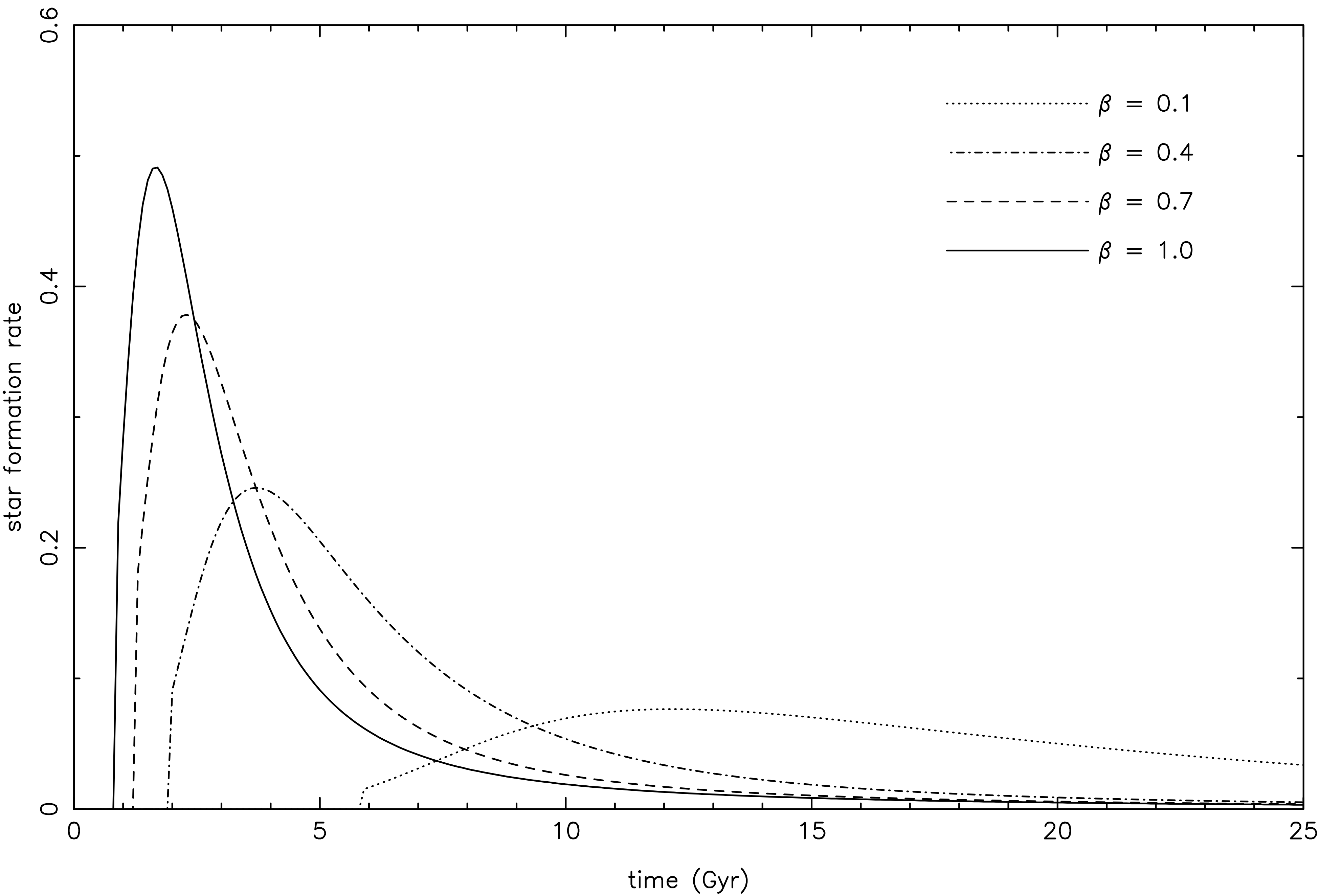
Table 1. The range of age predictions reported in the literature for the four methods of age determination considered in this paper

Constraint	Method	Age, T_0, (Gyr)	References
age of the solar neighbourhood	cooling of white dwarfs	9.3 ± 2	Winget et al. (1987)
age of the galactic disc	cosmochronology	10 - 30	Cowan, Thielemann & Truran (1991)
age of the galaxy	globular clusters	13 - 15 $\pm$ 3	Renzini (1993), Chaboyer (1995)
age of the universe	cosmological parameters	8 - 30	various, cf. van den Bergh (1994, 1995)

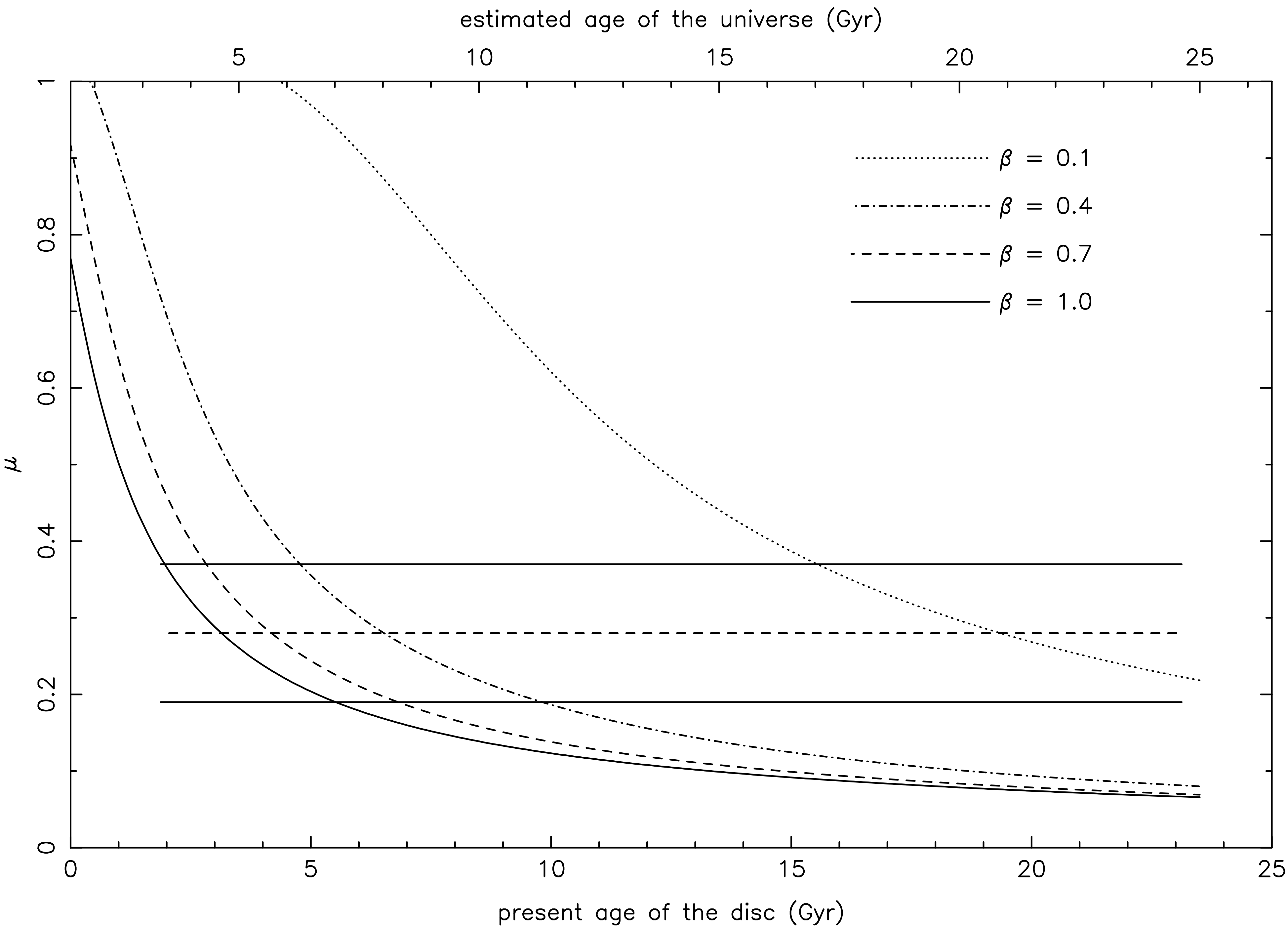
LIST OF FIGURES



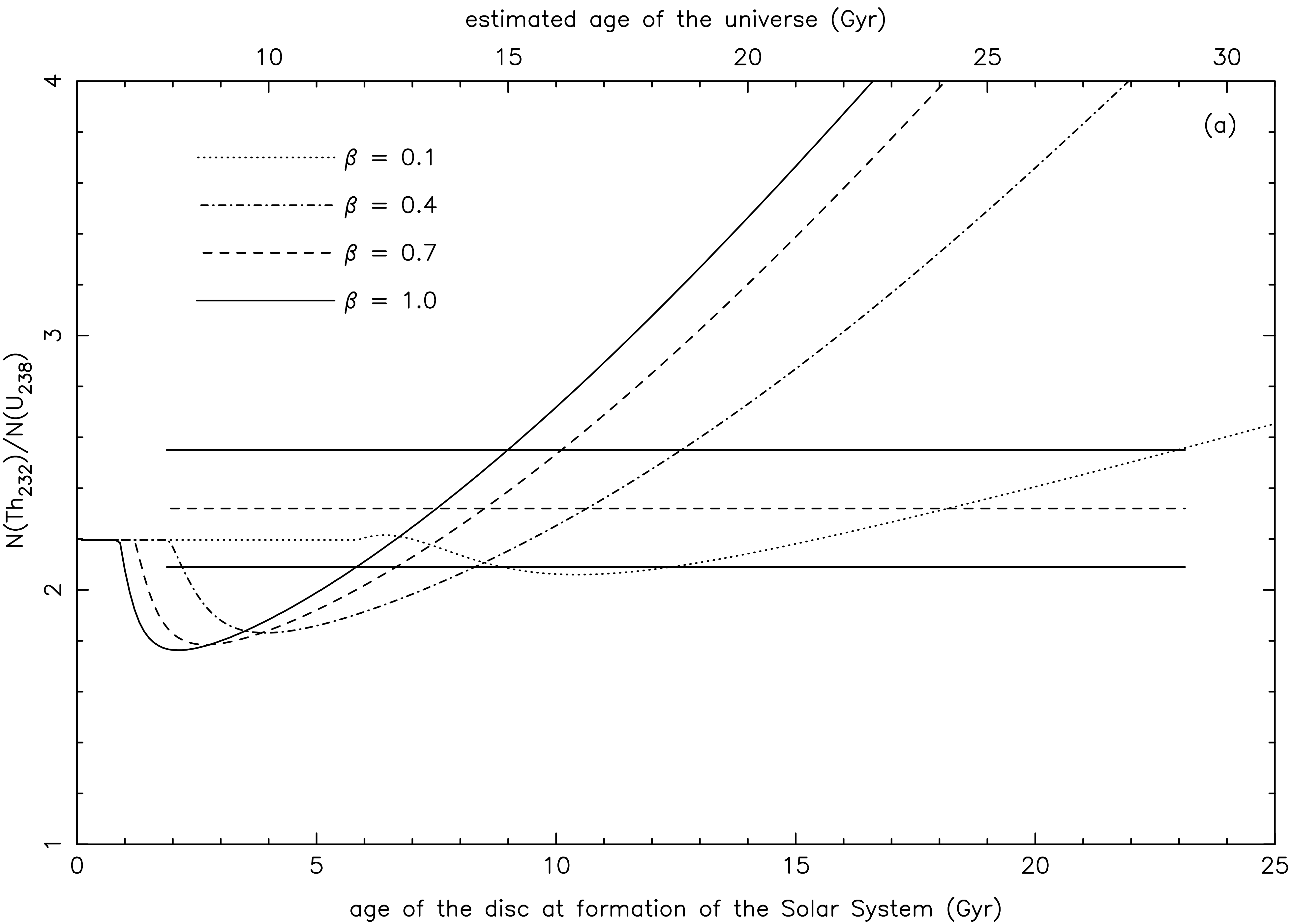
non−biased infall
biased infall
0
5
10
15
20
t* (Gyr)
R* (kpc)

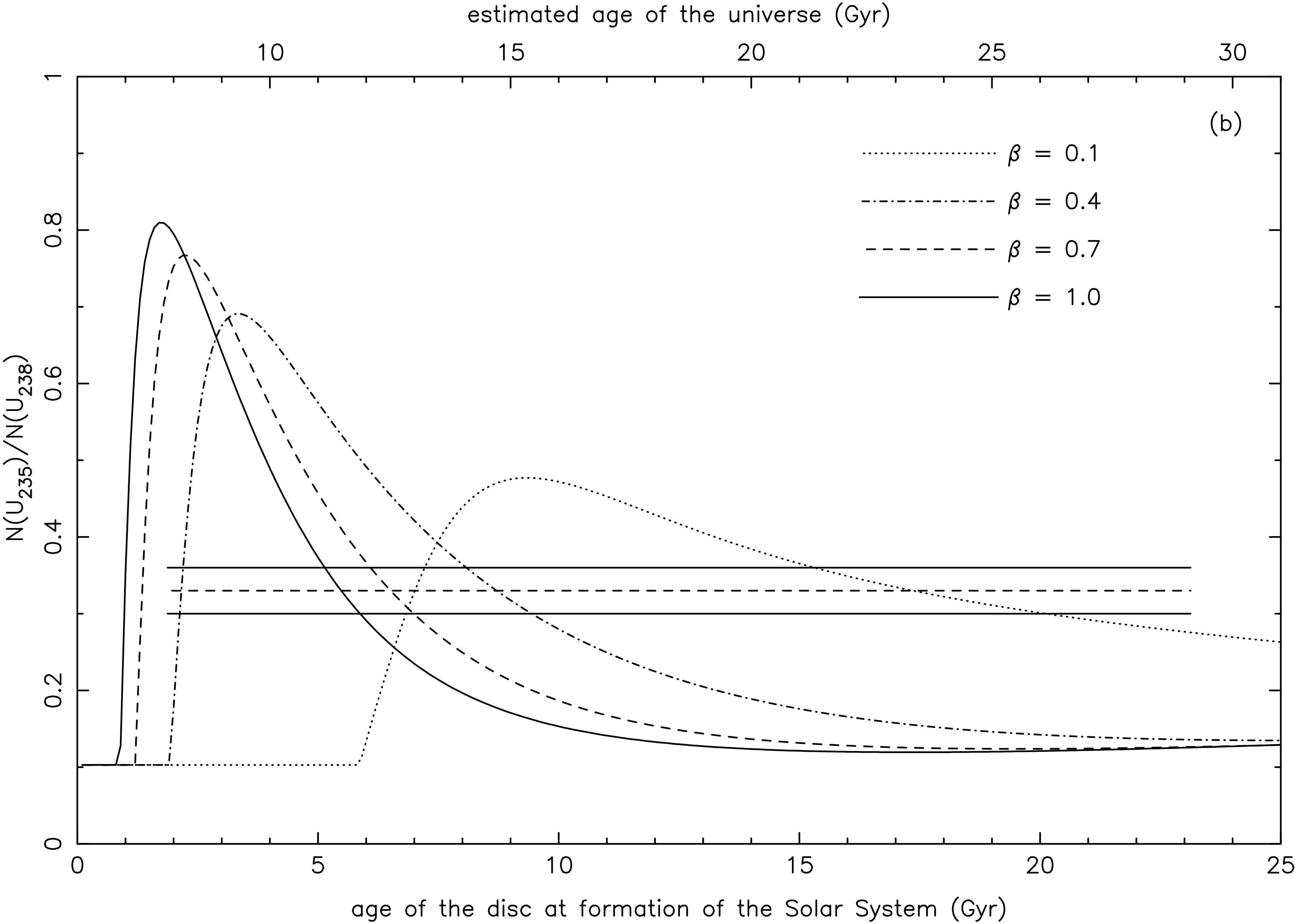


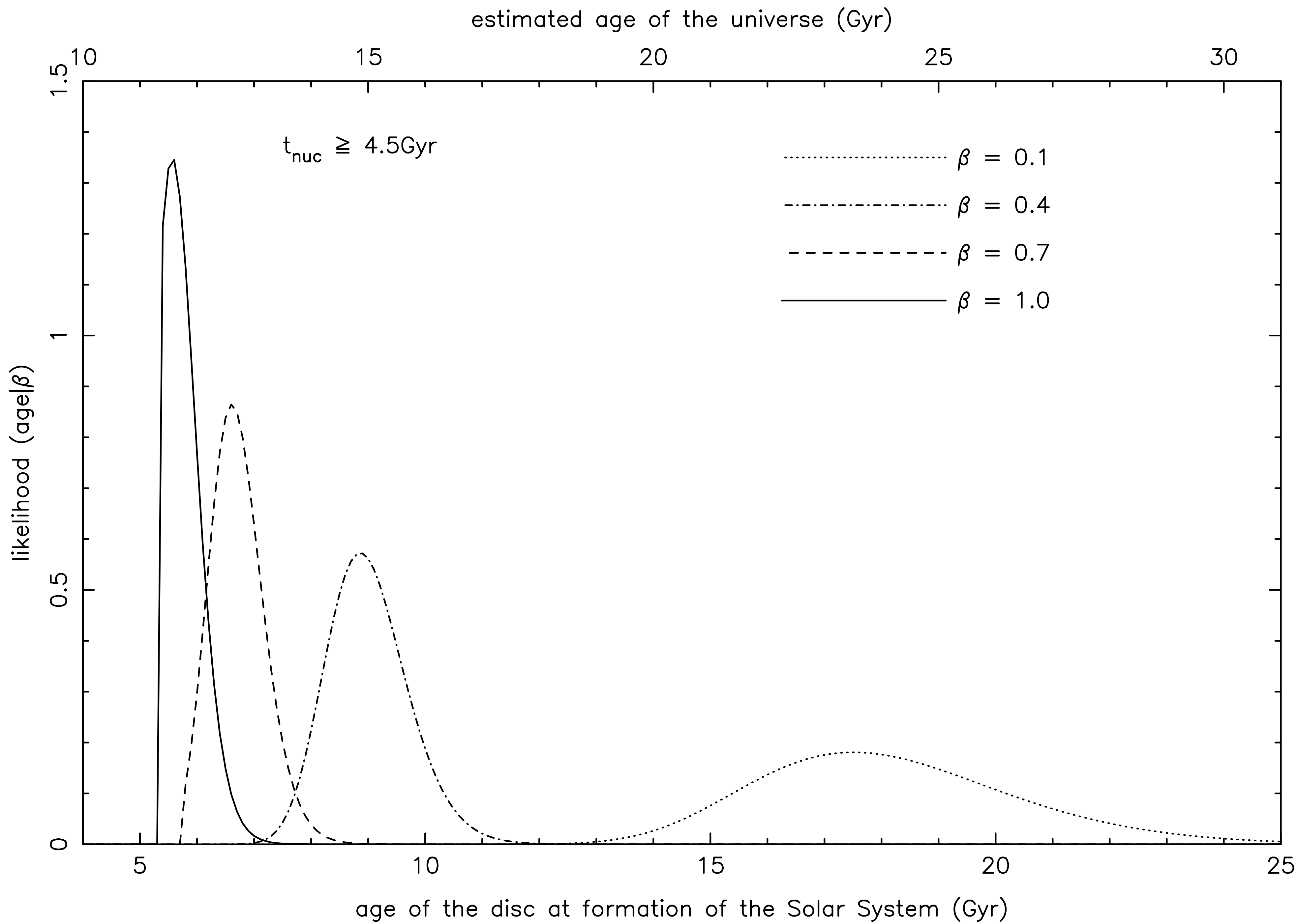
star formation rate
0.6
0.4
0.2
0
0
5
10
15
20
25
time (Gyr)
β = 0.1
β = 0.4
β = 0.7
β = 1.0

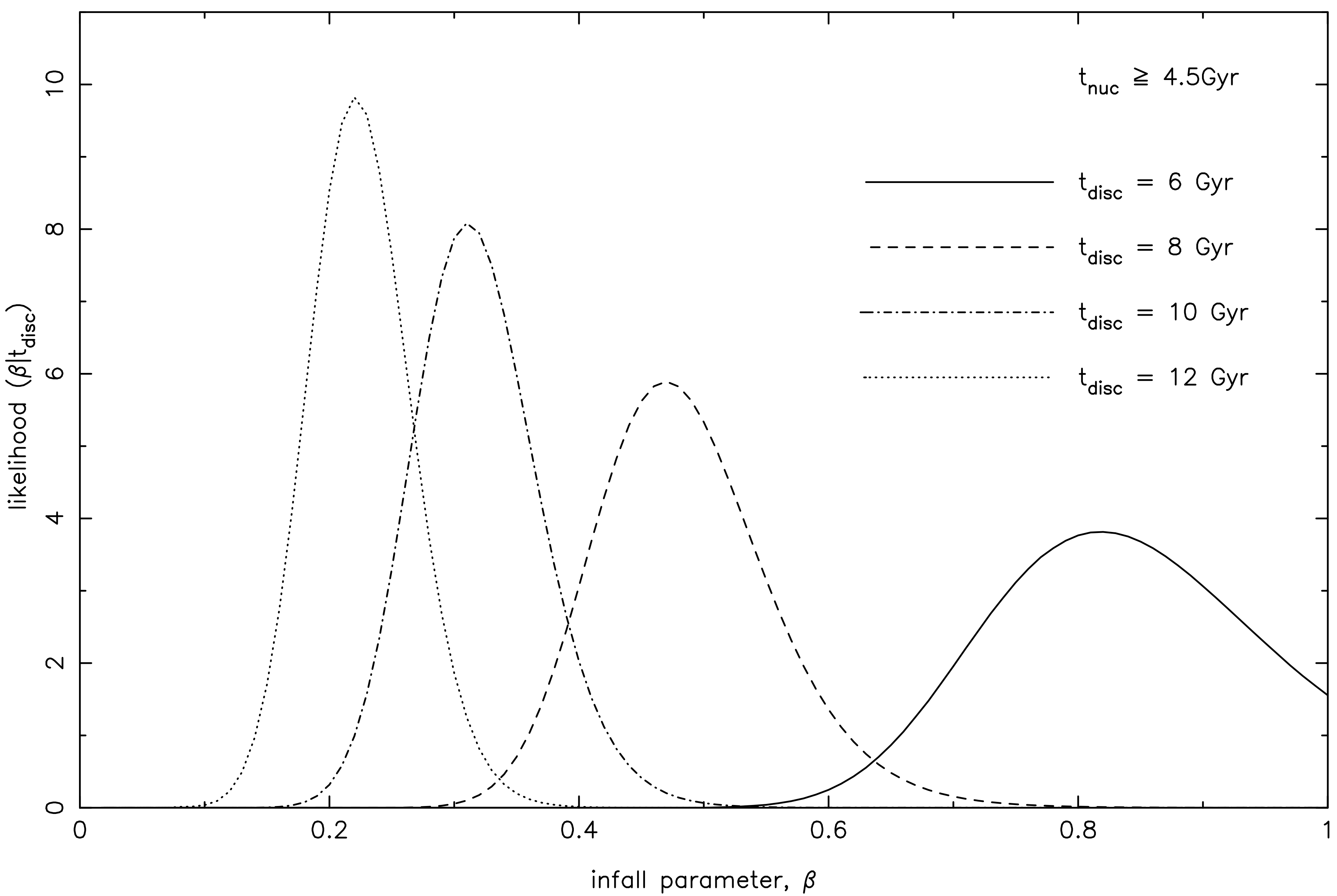


estimated age of the universe (Gyr)
present age of the disc (Gyr)
μ
β = 0.1
β = 0.4
β = 0.7
β = 1.0

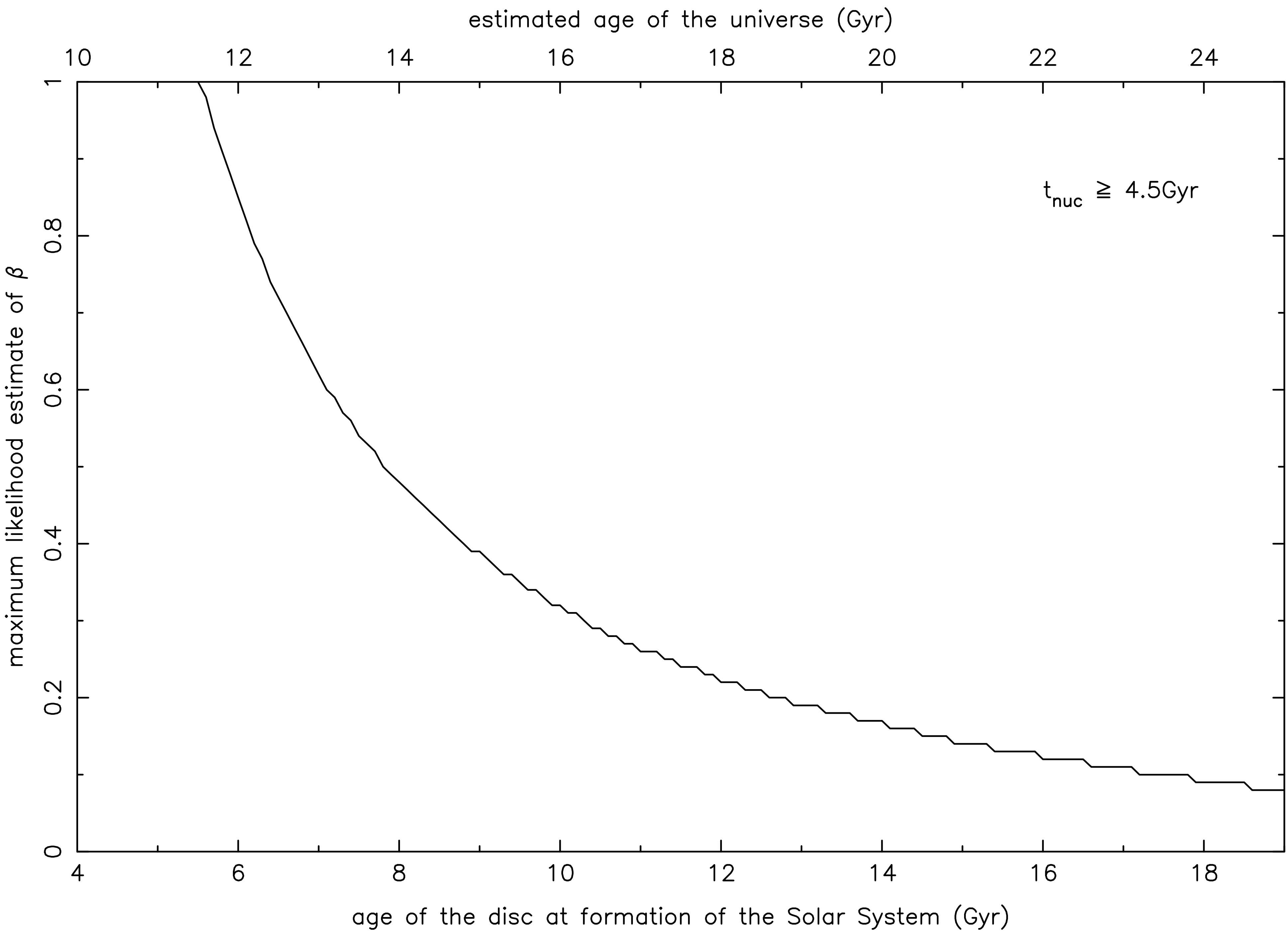

estimated age of the universe (Gyr)
10
15
20
25
30
(a)
β = 0.1
β = 0.4
β = 0.7
β = 1.0
N(Th232)/N(U238)
4
3
2
1
0
5
10
15
20
25
age of the disc at formation of the Solar System (Gyr)

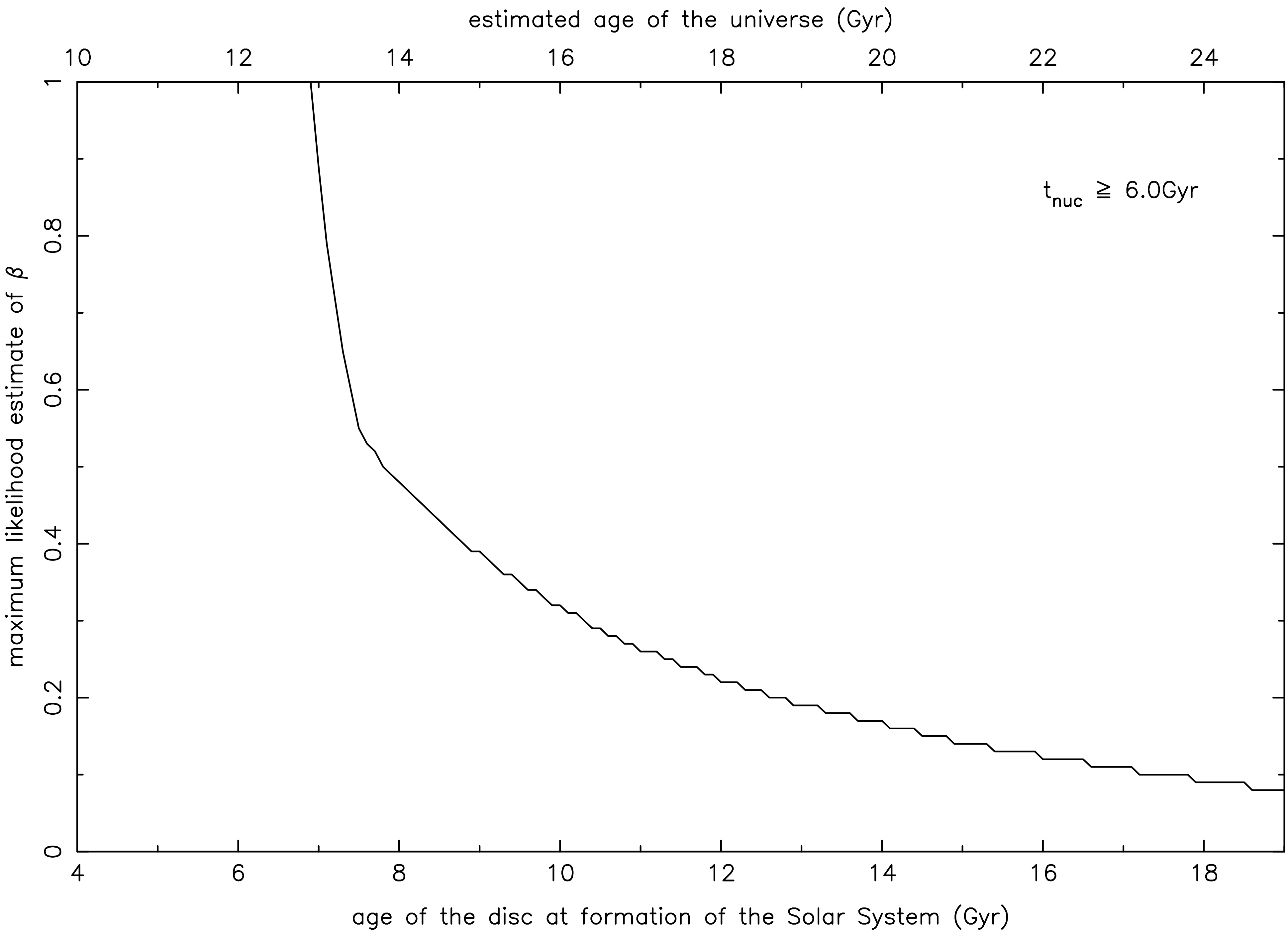

estimated age of the universe (Gyr)
10
15
20
25
30
(b)
β = 0.1
β = 0.4
β = 0.7
β = 1.0
$N(U_{235})/N(U_{238})$
0
0.2
0.4
0.6
0.8
1
0
5
10
15
20
25
age of the disc at formation of the Solar System (Gyr)


estimated age of the universe (Gyr)
10
15
20
25
30
1.5
1
0.5
0
likelihood (age|β)
t_nuc ≧ 4.5Gyr
β = 0.1
β = 0.4
β = 0.7
β = 1.0
5
10
15
20
25
age of the disc at formation of the Solar System (Gyr)

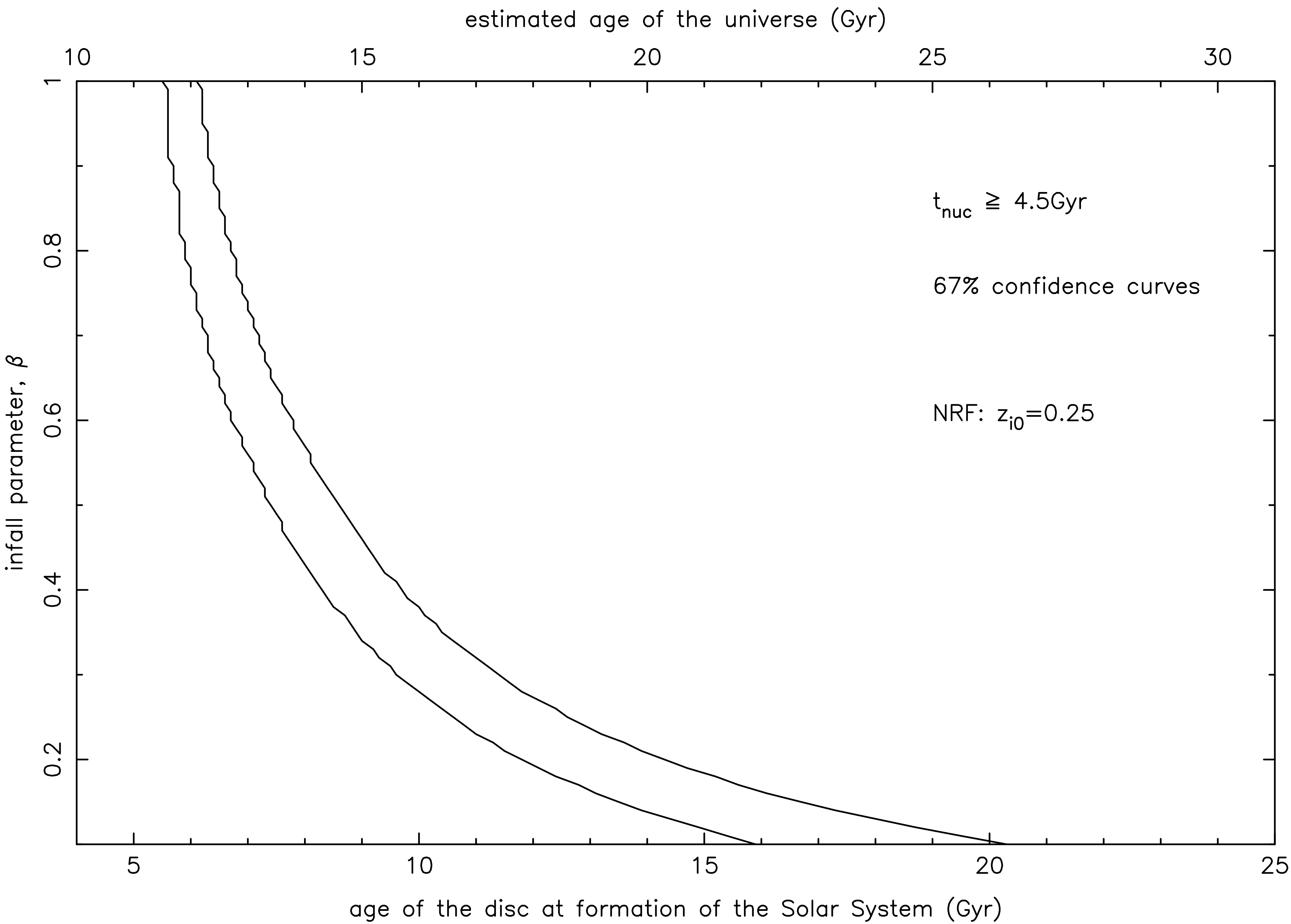


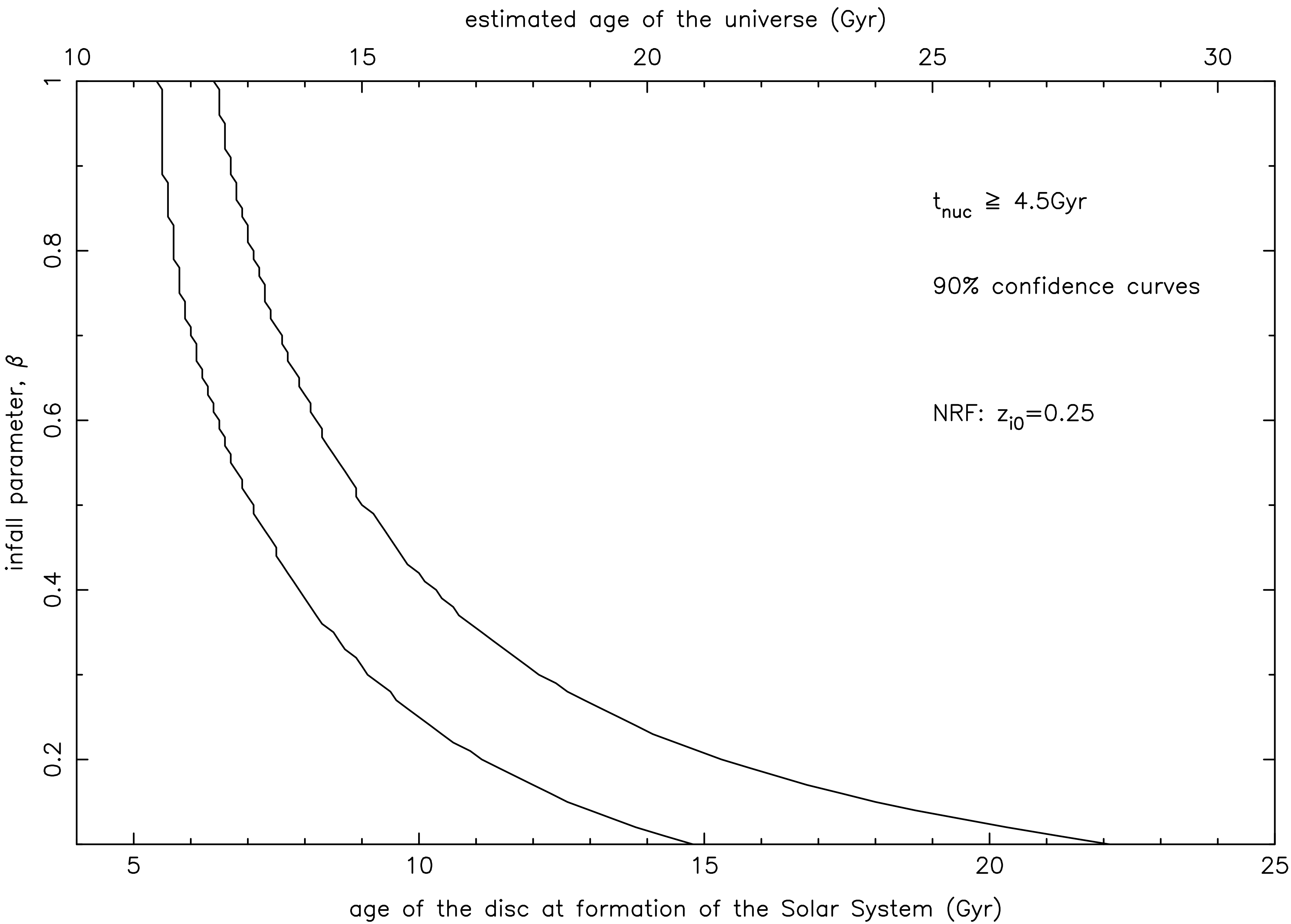
$t_{nuc} \geq 4.5$Gyr
t_{disc} = 6 Gyr
t_{disc} = 8 Gyr
t_{disc} = 10 Gyr
t_{disc} = 12 Gyr
likelihood ($\beta|t_{disc}$)
infall parameter, β
0
0.2
0.4
0.6
0.8
1
2
4
6
8
10


estimated age of the universe (Gyr)
10
12
14
16
18
20
22
24
maximum likelihood estimate of β
0
0.2
0.4
0.6
0.8
1
$t_{nuc} \geqq$ 4.5Gyr
4
6
8
10
12
14
16
18
age of the disc at formation of the Solar System (Gyr)



estimated age of the universe (Gyr)
10
12
14
16
18
20
22
24
maximum likelihood estimate of β
0
0.2
0.4
0.6
0.8
1
t_nuc ≧ 6.0Gyr
4
6
8
10
12
14
16
18
age of the disc at formation of the Solar System (Gyr)


estimated age of the universe (Gyr)
10
15
20
25
30
1
0.8
0.6
0.4
0.2
infall parameter, β
$t_{nuc} \geqq 4.5$Gyr
67% confidence curves
NRF: z_{i0}=0.25
5
10
15
20
25
age of the disc at formation of the Solar System (Gyr)



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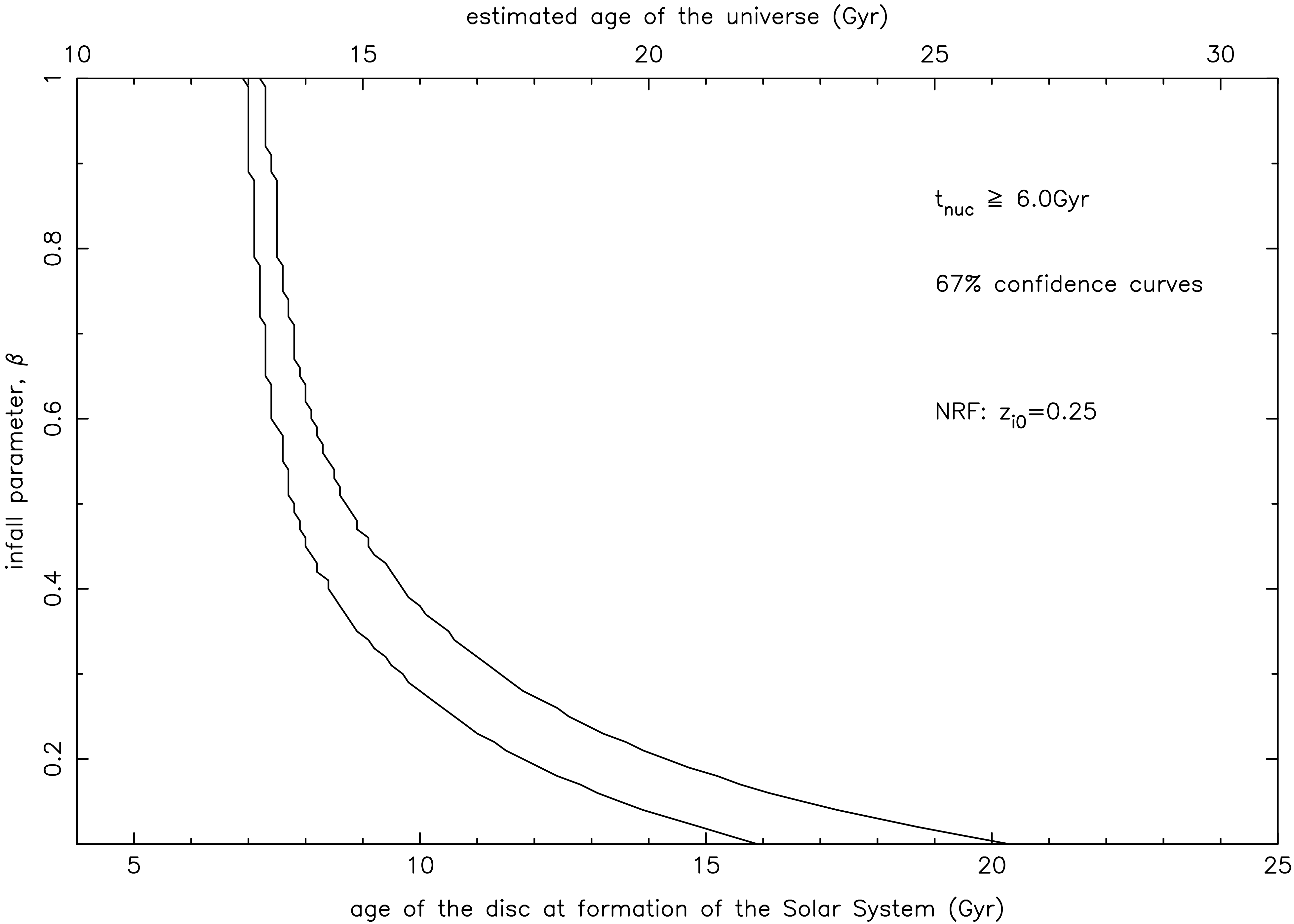
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infall parameter, β
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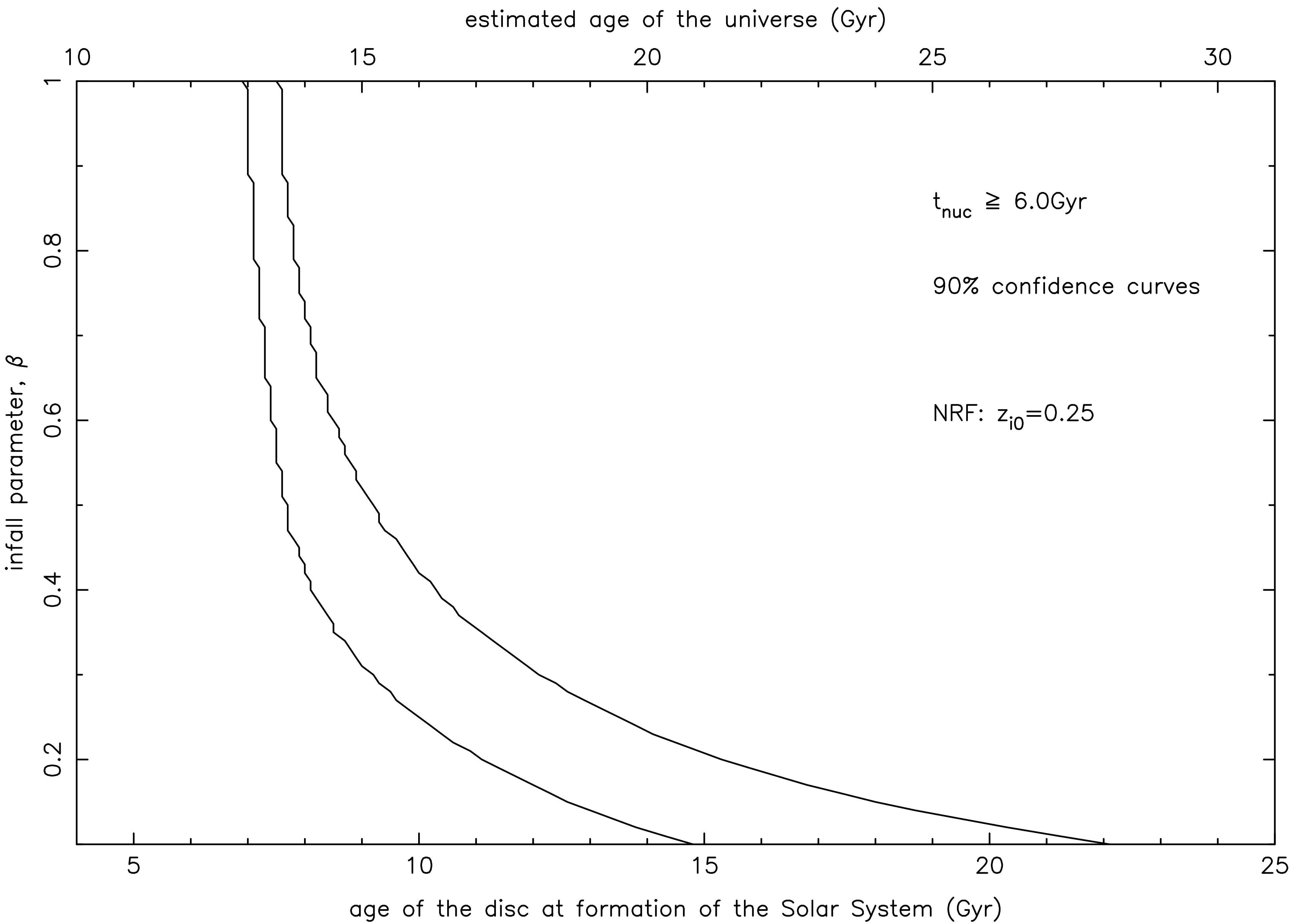

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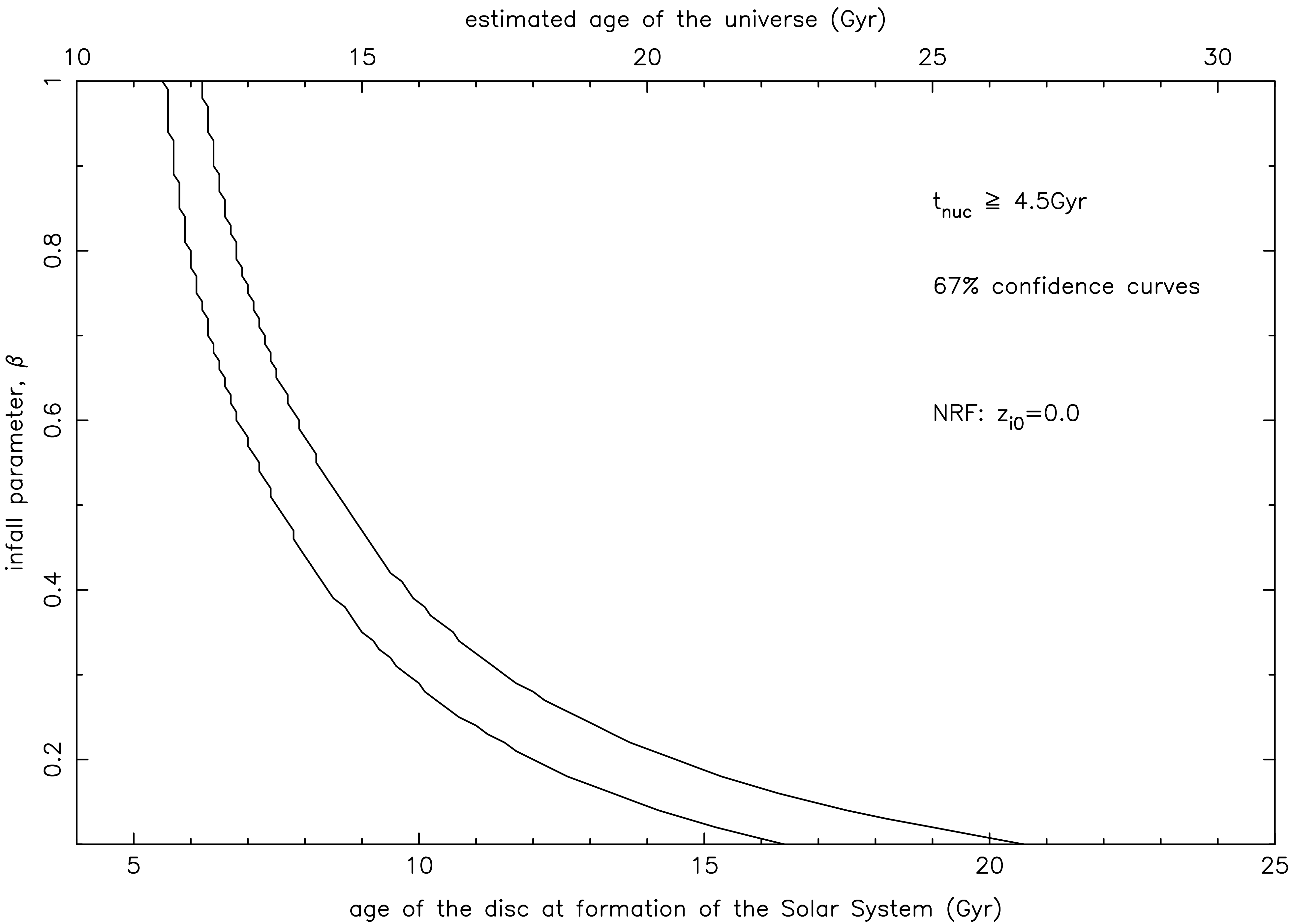
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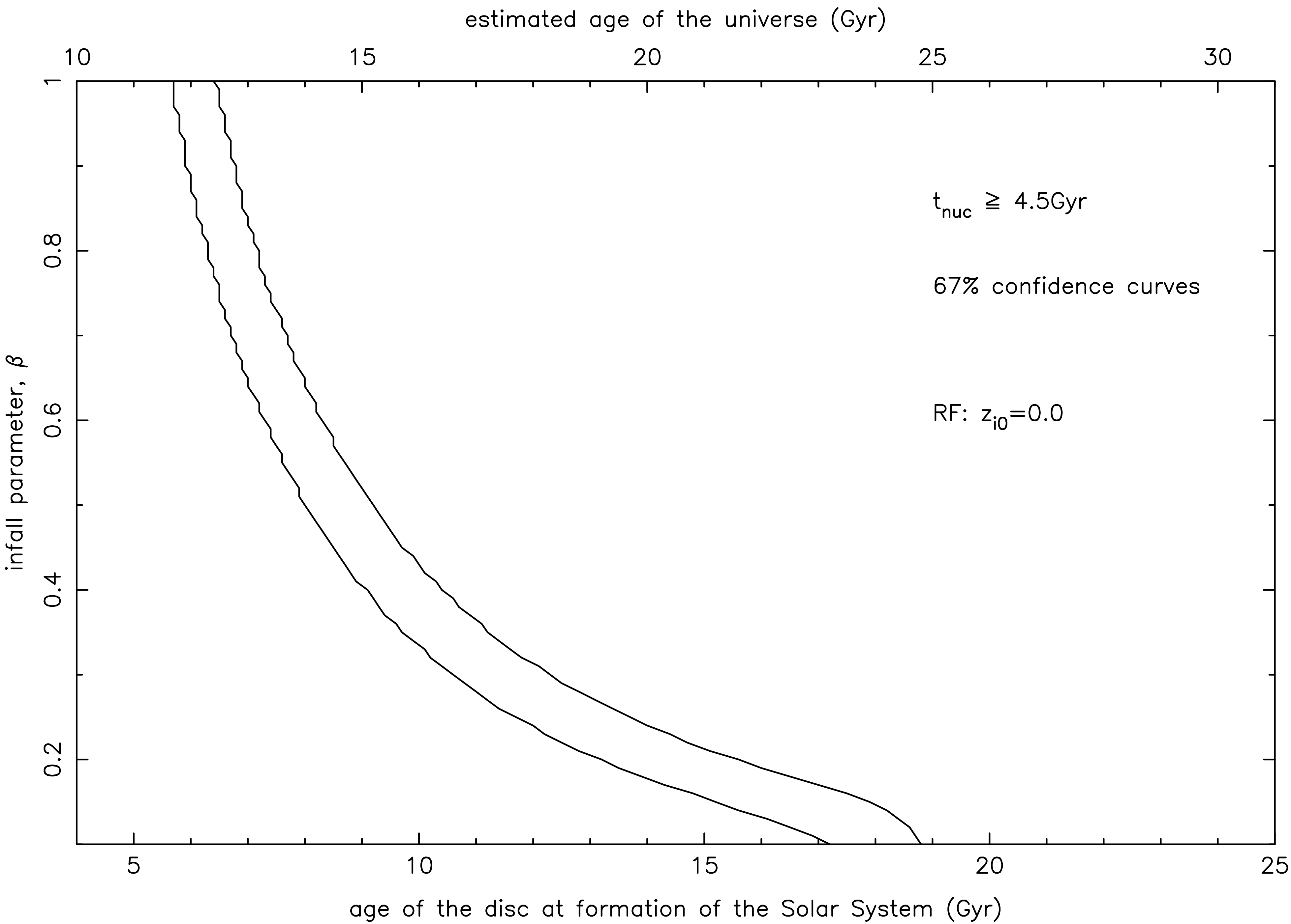
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